Tax Smoothing and the FTPL

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Motivation

- We started talking about nominal debt as shock absorber
- Now think of multiple assets, some real, some nominal
- Think about consequences of insurance

New element: distortionary taxation, government spending

- Retain taxes as lump sum
- Introduce distortion as administrative cost
- Slight change in timing, taxes are now a "credit good," pay at the beginning of period t $\tau_{t-1} P_{t-1}$
- To get $\tau_t = T_t / P_t$ funds you need to tax households by $f(\tau_t) P_t = (\tau_t + \gamma \tau_t^2 / 2) P_t$
- Stochastic process for government spending, G_t

New element: real debt

- Gov't trades real bonds
- Denoted by b_{t-1}
- Real bonds defaultable (haircut δ_t)
- Assume ϵ cost of default (will use as a tie breaker, not keep track of ϵ)

Endogenous policy

- Study Ramsey problem:
  - Compute set of competitive equilibria
  - Identify the best
- Will talk about sovereign default, but we have commitment

Household budget constraint

- Still use cashless limit, just to save a bit of notation
- Still want cash in the background
- $$B_{t-1} + P_t b_{t-1}(1-\delta_t) + P_{t-1}(y - c_{1\,t-1} - c_{2\,t-1} - \tau_{t-1} - \tau_{t-1}^2/2)$$
  $$+ A_t \geq \frac{B_t}{1+R_t} + \frac{b_t P_t}{1+r_t} + E_t(z_{t,t+1}A_{t+1})$$

Government budget constraint

- $$B^S_{t-1} + b^S_{t-1} P_t (1-\delta_t) - (\tau_{t-1} - G_{t-1}) P_{t-1} = \frac{B^S_t}{1+R_t} + \frac{b^S_t P_t}{1+r_t}$$
- Define
  $$W_t := B^S_{t-1} + b^S_{t-1} P_t (1-\delta_t) - (\tau_{t-1} - G_{t-1}) P_{t-1}$$

Key competitive equilibrium conditions: Euler equations

- $$1 = E_t \left[\beta (1 + R_{t+1}) \frac{P_t}{P_{t+1}} \right] \quad t \geq 0$$

- $$\frac{1}{1 + r_t} = \frac{\beta E_t[(1 + R_{t+1})(1 - \delta_{t+1})]}{1 + R_t}$$

Key competitive equilibrium conditions: gov't PVBC

$$W_t = \sum_{s=0}^{\infty} E_t \left[z_{0,t+s+1} \left(\tau_{t+s} - G_{t+s} \right) P_s \right]$$

Also known as "government valuation equation"

Computing household welfare

- Neglect consumption of cash good (just for simplicity)
- From market clearing

$$c_{2,t-1} = y - G_{t-1} - \tau_{t-1}^2/2$$

- Household welfare is

$$E_0 \sum_{t=0}^{\infty} \beta^t [y - G_t - \tau_t^2/2]$$

Relaxed implementability constraint

- Take household first-order conditions and replace $z_{0,t+1}$ and P_t into the Gov't PVBC
- Why is this a relaxed problem?

Relaxed implementability constraint

- Take household first-order conditions and replace $z_{0,t+1}$ and P_t into the Gov't PVBC
- Why is this a relaxed problem?
- Because we have not established that markets are complete
- It's optimal policy that will make them complete
- Obtain

$$W_0 = \frac{P_0}{1 + R_0} E_0 \sum_{t=0}^{\infty} \beta^t (\tau_t - G_t)$$

Ramsey problem

$$\min \sum_{t=0}^{\infty} \beta^t \tau_t^2/2$$

subject to

$$E_0 \sum_{t=0}^{\infty} \beta^t \tau_t = \frac{W_0(1+R_0)}{P_0} + E_0 \sum_{t=0}^{\infty} \beta^t G_t$$

- Take P_0 as given (usual problem of taxation of initial wealth)
- Solution: $\tau_t = \bar{\tau}$ (level determined by BC)

Implementation of the Ramsey solution

$$\frac{\bar{\tau}}{1-\beta} = \frac{W_t(1+R_t)}{P_t} + E_t \sum_{s=0}^{\infty} \beta^s G_{t+s}$$

- All innovations in the PV of spending must be absorbed by $W_t(1+R_t)/P_t$
- Playing with R_t involves distortions on cash goods, not desirable
- Study W_t/P_t

Using inflation or default

$$\frac{W_t}{P_t} := \frac{B^S_{t-1}}{P_t} + b^S_{t-1}(1-\delta_t) - (\bar{\tau} - G_{t-1})\frac{P_{t-1}}{P_t}$$

- First option: inflation
- Second option: default (remember ϵ)

More insights

$$\frac{W_t}{P_t} := \frac{B^S_{t-1}}{P_t} + b^S_{t-1}(1-\delta_t) - (\bar{\tau} - G_{t-1})\frac{P_{t-1}}{P_t}$$

- The more nominal debt there is, the smaller inflation needs to be
- If nominal debt is too small, nonnegativity of $\frac{B^S_{t-1}}{P_t}$ binds, default kicks in
- Riskiness of nominal debt higher with less nominal debt
- At some point, riskiness of real debt kicks in

Generalizing results

- What if inflation (and default) are costly?
- ... then, move taxes around somewhat...
- ... similar insight, even more important to have large nominal debt.
- Now, deficits in the short run, surpluses in the long run
- Econometric issues in Cochrane (2001)