A Cash-in-Advance Economy

Marco Bassetto

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Our Lab Economy

- Will need a common framework to think about several distinct policies
- Will build from explicit microfoundations, to understand incentives
- Want to talk about monetary/fiscal policy, will need:
  - A motive for the existence of money
  - Taxes
  - Government bonds

Notable elements from which we abstract

- Heterogeneity
- Nominal frictions (e.g., Phillips curve)

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- Heterogeneity
- Nominal frictions (e.g., Phillips curve)
- Are these elements important?
- For some questions, very important (e.g. cost of inflation, price impact of money increase)
- ... but not for those that we will ask:
    - Are monetary and fiscal policy connected?
    - How is the price level determined?

Suggested reading

- My manuscript (Bassetto, 2017) covers the deterministic case
- Woodford (1994): greater depth on money supply rules + basic FTPL

Our lab economy: commodity space

- Time: discrete and infinite: $t = 0, 1, 2, \ldots;$
- Large number ("continuum") of identical "islands;"
- A single good per period and island;
- "Money," a useless object

Agents

- Large number ("continuum") of identical households;
- Monetary authority (CB, for "central bank")
- Fiscal authority ("Treasury")
- CB + Treasury = "government"

Household preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_{1t}) + c_{2t}] \tag{1}$$

- c_{1t} and c_{2t} will be acquired from different islands;
- We will call goods 1 "cash goods" and goods 2 "credit goods"
- Notice special assumption: quasilinear preferences (linear in credit goods)
- Assume also $\lim_{c_{1t} \to 0} u'(c_{1t}) = \infty$, $\lim_{c_{1t} \to \infty} u'(c_{1t}) = 0$ ("Inada" conditions)
- Last two assumptions simplify the algebra

Government behavior

- Will not describe what motivates government
- Will consider various strategies, implications

Technology, markets, information: First part of period

- A sunspot shock s_t and a potential fiscal shock τ_t are realized
- Each household has a home island, where it starts each period with y_t units of the good; the good cannot be stored
- Money can be produced for free by the monetary authorities, perfectly storable
- Asset markets open:
  - households and government trade money and nominally risk-free debt,
  - households trade state-contingent debt (in zero net supply),
  - Treasury levies taxes (in money): T_t
- Note: time-t variables are adapted to $\{s_s, y_s, \tau_s\}_{s=0}^{t}$.

Technology, markets, information: Second part of period

- Households divide: shopper, worker
- Worker stays on island, sells goods to others, buys credit goods
- Can promise to settle payment at beginning of next period
- Shopper travels to another island where she is anonymous
- Shopper buys cash good, but needs money

Household flow budget constraint

$$B_{t-1} + M_{t-1} + P_{t-1}(y - c_{1\,t-1} - c_{2\,t-1}) + A_t - T_t \geq \frac{B_t}{1+R_t} + M_t + E_t(z_{t,t+1}A_{t+1}) \qquad (2)$$

- M_t: (nominal) money holdings (currency unit: "dollar")
- B_t: (nominal) debt (number of dollars promised at t to be paid at $t+1$)
- A_{t+1}: (nominal) state-contingent debt (number of dollars promised at t to be paid at $t+1$, contingent on $t+1$ shocks)
- P_t: price level in period t
- R_t: nominal interest rate between period t and $t+1$
- $z_{t,t+1}$: state-price deflator between periods t and $t+1$

Some convenient definitions

- Nominal wealth coming into the period:

$$W_t := B_{t-1} + M_{t-1} + P_{t-1}(y - c_{1\,t-1} - c_{2\,t-1}) + A_t - T_t$$

- Multi-period asset-pricing kernel:

$$z_{0,0} := 1, z_{0,t} := \prod_{s=0}^{t-1} z_{s,s+1}, t > 0$$

$$z_{t,s} := z_{0,s} / z_{0,t}, s \geq t$$

- Will take W_0 as an exogenous initial condition

Cash-in-Advance Constraint

$$M_t \geq P_t c_{1t} \tag{3}$$

Note: it implies $M_t \geq 0$

No-Ponzi condition

$$W_t \geq -\limsup_{n\to\infty} \sum_{s=t}^{n} E_t[z_{t,s+1}(P_s y_s - T_{s+1})]$$

Will mostly consider "sane" policies where we can replace the limsup with a regular limit.

Government budget constraint

$$B_{t-1}^S + M_{t-1}^S - T_t = \frac{B_t^S}{1+R_t} + M_t^S$$

- B_t^S: bonds supplied by government
- M_t^S: money supplied by government

Does gov't have a no-Ponzi condition?

Equilibrium concept: Competitive equilibrium: Elements

- An allocation $(c_{1t}, c_{2t}, M_t, B_t, A_{t+1})_{t=0}^{\infty}$
- A price system $(P_t, R_t, z_{t,t+1})_{t=0}^{\infty}$
- A government policy $(T_t, B_t^S, M_t^S)_{t=0}^{\infty}$

Equilibrium concept: Competitive equilibrium: Requirements

- Allocation maximizes household utility subject to their budget constraint, cash-in-advance, and no-Ponzi, taking gov't policy and prices as given (notice rational expectations);
- Markets clear:

$$B_t = B_t^S, M_t = M_t^S, c_{1t} + c_{2t} = y_t, A_{t+1} \equiv 0$$

- Gov't budget constraint is met period by period

Household Lagrangean

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ u(c_{1t}) + c_{2t} +$$

$$\lambda_t \Big[B_{t-1} + M_{t-1} + A_t + P_{t-1}(y - c_{1\,t-1} - c_{2\,t-1})$$

$$- T_t - \frac{B_t}{1+R_t} - M_t - E_t(z_{t,t+1} A_{t+1}) \Big] + \mu_t (M_t - P_t c_{1t}) \Big\}$$

- Bewley, *Journal of Economic Theory*, 1972
- Luenberger, Optimization by Vector Space Methods, 1969
- Stokey, Lucas, and Prescott (dealing with the issue of LM being in L_1)

First-order conditions

First-order conditions:

- Cash goods:

$$u'(c_{1t}) = (\beta E_t \lambda_{t+1} + \mu_t) P_t, \quad t \geq 0$$

- Credit goods:

$$1 = \beta E_t \lambda_{t+1} P_t, \quad t \geq 0$$

- Money:

$$\lambda_t = \mu_t + \beta E_t \lambda_{t+1}, \quad t \geq 0$$

- Government Bonds:

$$\frac{\lambda_t}{1 + R_t} = \beta E_t \lambda_{t+1}, \quad t \geq 0$$

- State-Contingent Bonds:

$$\lambda_t z_{t,t+1} = \beta \lambda_{t+1}, \quad t \geq 0$$

Transversality condition

One more necessary condition ($t \geq 0$):

$$E_t\left[\liminf_{T\to\infty} \beta^T \left[\lambda_T\left(W_T - \limsup_{n\to\infty} \sum_{s=T}^{n} E_t[z_{T,s+1}(P_s y_s - T_{s+1})]\right)\right]\right] = 0$$

- Weitzman, *Management Science*, 1973 (deterministic case)
- Coșar and Green, *Macroeconomic Dynamics*, 2016 (stochastic case)

The Friedman distortion

$$u'(c_{1t}) = 1 + R_t, \quad t \geq 0 \tag{4}$$

$$R_t > 0 \Longrightarrow M_t = P_t c_{1t}, \quad t \geq 0 \tag{5}$$

- Money is free to produce
- When $R_t > 1$, gov't charges for it
- Consumption tilted towards credit
- Only source of inflation cost in this model
- Cost related to expected inflation

The Fisher equation

$$1 = E_t \left[\beta (1 + R_{t+1}) \frac{P_t}{P_{t+1}} \right] \quad t \geq 0 \tag{6}$$

- Positive relation between interest rates and inflation
- Holds across most models (and data) in the long run
- Here, in the short run as well

Gov't policy regime

- Description of competitive equilibrium cannot be completed without knowing what gov't does
- Will explore various options

A pure money supply rule

- Set $B_t^S = 0$;
- Set $M_t^S = (1+q)M_{t-1}^S$;
- Set $T_t = M_{t-1}^S - M_t^S$.

Equilibria under a money supply rule: the log case

Suppose $u(c_{1t}) = \log c_{1t}$, $q > \beta - 1$, use (4), (5), and (6):

- $$c_{1t} = \frac{\beta M_t}{M_{t+1}} = \frac{\beta}{1+q}, \quad t \geq 0$$
- $$\frac{P_{t+1}}{P_t} = \frac{M_{t+1}}{M_t} = 1 + q, \quad t \geq 0$$
- $$P_0 = \frac{M_0(1+q)}{\beta}$$
- Unique equilibrium, price level pinned down

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- More problems:
  - Fiscal policy is more important than it seems
  - Works fine for log, but what about other preferences?

Equilibria under a money supply rule: other power cases

- Now $u(c_{1t}) = c_{1t}^{1-\sigma}/(1-\sigma)$.
- To keep it simple, set $q = 0$: $M_t = M_{t-1} = \bar{M}$, and study deterministic equilibria.

Use again (4), (5), and (6):

$$\bar{M}^{-\sigma} = E_{t-1} \frac{\beta P_{t-1}}{P_t^{1-\sigma}}, \quad t \geq 1$$

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so (for deterministic equilibria)

$$\log P_t = \left(\frac{1}{1-\sigma}\right)^t \left[\log P_0 - \log(\bar{M}\beta^{-\frac{1}{\sigma}})\right] + \log(\bar{M}\beta^{-\frac{1}{\sigma}}), \quad t \geq 1$$

Unique SS at $P = \bar{M}\beta^{-\frac{1}{\sigma}}$

Deterministic equilibria under a money supply rule: $\sigma > 2$

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- Name any $P_0 > 0$, solve for sequence that converges to SS

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- ... but at least it converges back to SS?

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- Name any $P_0 > 0$, solve for sequence that converges to SS
- Disturbing: price level can be anything
- ... but at least it converges back to SS?
- With randomness, it could bounce away from SS all the time

Deterministic equilibria under a money supply rule: $\sigma \leq 2$

- For $\sigma = 2$, we get a period-2 cycle outside of SS
- For $1 < \sigma < 2$, we get paths where prices oscillate, exploding to $+\infty$ every other period and going to 0 every other period
- For $\sigma < 1$, we get either prices going to $+\infty$ or to 0
- Are these equilibria?
- One more condition to check: transversality condition

Checking the transversality condition

Use (4), (5), and (6) once more, get

$$\prod_{s=0}^{t}(1+R_s)^{-1} = (1+R_0)^{-1}\beta^t \frac{P_0}{P_t}$$

Substitute into transversality condition, use constant money supply rule; transv. will hold if

$$\bar{M}(1+R_0)^{-1}P_0 \liminf_{t\to\infty} \frac{\beta^t}{P_t} = 0$$

Take logs, use difference equation: transv. will hold if

$$\liminf_{t\to\infty}\left[t\log\beta - (1-\sigma)^{-t}\left(\log P_0 - \log\left(\bar{M}\beta^{-\frac{1}{\sigma}}\right)\right)\right] + \log\left(\bar{M}\beta^{-\frac{1}{\sigma}}\right) = -\infty.$$

Conclusion from checking transversality condition

- If $\sigma = 2$, bounded oscillations, OK
- If $1 < \sigma < 2$, explosive oscillations (converging to 0 in either period and to $+\infty$ every other period), OK according to current transv
- If $\sigma < 1$, prices blow up to ∞ or go down to 0, transv. OK only when they go to ∞

Bottom line on money supply rules

When $\sigma < 1$ and $q \geq 0$ (most plausible case):

- SS equilibrium
- Continuum of equilibria where P_0 starts above SS, $P_t \to \infty$, so $c_{1t} = \bar{M}/P_t \to 0$
- No guarantee that money will have value!

A different way of running monetary policy

- Since the mid-1980s, central banks only use interest rates as the main policy tool
- Caveat: until the great recession, they did manage money on a day-to-day basis
- 2nd caveat: they now use QE too ("bonds-in-advance?")

The simplest case: A fixed interest rate peg

- New specification of monetary-fiscal policy:
- Central bank sets $R_t = \bar{R}$ in every period.
- Gov't budget constraint:

$$B_{t-1}^S + M_{t-1}^S - T_t = \frac{B_t^S}{1+\bar{R}} + M_t^S$$

Infinitely elastic supply for M_t^S and B_t^S (but sum of two is set)

- Taxes: extremely important, but we will talk about them later

Computing the set of equilibria under an interest rate peg

- Friedman distortion:

$$u'(c_{1t}) = 1 + \bar{R} \implies \text{ get } c_{1t}!$$

- Fisher equation:

$$1 = \beta(1 + \bar{R})E_t\left(\frac{P_t}{P_{t+1}}\right)$$

- Cash-in-advance (if $\bar{R} > 0$): $M_t = P_t c_{1t}$

The good news

- The real allocation (consumption of cash vs. credit goods) pinned down
- Expected (inverse) inflation pinned down

The bad news

- For now, no way to pin down P_0
- Why is this?

$$M_0 = P_0 c_{10}$$

  and $M_0 = M_0^S$, but M_0^S can be anything
- Price level is "indeterminate"
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  ... what about our old friend, the transversality condition?
  [Suspense]

More complicated rules

- So far, interest rate was unconditional commitment
- What if the interest rate responds to the past?
- E.g., to past inflation?
- Note: look only at deterministic equilibria, but we have sunspot equilibria every time there are multiple deterministic equilibria

Taylor rules

- Consider

$$R_{t+1} = \frac{\bar{\pi}^{1-\alpha}}{\beta} \left(\frac{P_t}{P_{t-1}} \right)^{\alpha} - 1, \qquad (7)$$

where $\bar{\pi}$ is some target inflation rate

- Define $\pi_t := P_t / P_{t-1}$. Substitute (7) into Fisher equation:

$$\pi_{t+1} = \bar{\pi}^{1-\alpha} (\pi_t)^{\alpha}.$$

- Steady state:

$$\pi^{SS} = \bar{\pi}^{1-\alpha} \left(\pi^{SS} \right)^{\alpha} \implies \quad \pi^{SS} = \bar{\pi}$$

A digression on names

- Purists call a rule that links interest rates to other variables a "Wicksell" rule
- A Taylor rule is reserved for the case in which interest rates respond to inflation with a coefficient of 1.5 and to output with a coefficient of 0.5
- We will use Taylor as a name throughout

The Taylor principle

- Are there equilibria outside of SS?
- Take logs:

$$(\log \pi_{t+1} - \log \bar{\pi}) = \alpha(\log \pi_t - \log \bar{\pi})$$

- If $|\alpha| < 1$, many equilibria converging to SS (local indeterminacy): name P_0, get R_1, which affects P_1 and so on
- If $|\alpha| > 1$, SS is locally unique: any path that starts away from SS will move further away, local determinacy, success?

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- If $|\alpha| > 1$, SS is locally unique: any path that starts away from SS will move further away, local determinacy, success?
- Is there something wrong with paths where $\log \pi$ diverges?

Stochastic equilibria

Define

$$q_{0,t} := E_0\left[\frac{1}{\pi_t}\right]$$

With the Taylor rule, get

$$\log q_{0,t+1} = (\alpha - 1)\log \pi + \alpha \log q_{0,t}$$

Similar explosion unless the initial condition is just right.

Taylor rules and the zero lower bound (ZLB)

- Money pays zero interest rate
- Bonds cannot pay negative interest rates
- Need to check $R_t \geq 0$:

$$\frac{\bar{\pi}^{1-\alpha}}{\beta}\left(\frac{P_t}{P_{t-1}}\right)^{\alpha} > 1$$

or

$$\log\left(\frac{\bar{\pi}}{\beta}\right) + \alpha(\log \pi_t - \log \bar{\pi}) > 0$$

- Need $\bar{\pi} > \beta$ for SS
- If π_t sufficiently low, ZLB is hit

The perils of Taylor rules (Benhabib, Schmitt-Grohé, and Uribe)

- Amended Taylor rule:

$$R_{t+1} = \max\{0, \frac{\bar{\pi}^{1-\alpha}}{\beta}(\pi_t)^{\alpha} - 1\}, \tag{8}$$

- New difference equation:

$$\pi_{t+1} = \max\{\beta, \bar{\pi}^{1-\alpha}(\pi_t)^{\alpha}\}$$

- Is β new SS? Check whether ZLB is hit at $\pi^{SS} = \beta$:

$$\frac{\bar{\pi}^{1-\alpha}}{\beta}(\beta)^{\alpha} < 1 \iff \left(\frac{\bar{\pi}}{\beta}\right)^{1-\alpha} < 1 \iff \alpha > 1$$

- So, same condition that yields local determinacy also yields new SS, locally indeterminate
- Bottom line: no, this is not success, we still have lots of candidate equilibria

Playing with the timing of Taylor rules

- What if

$$R_{t+1} = \frac{\bar{\pi}^{1-\alpha}}{\beta} \left(\frac{P_{t+1}}{P_t} \right)^{\alpha} - 1? \tag{9}$$

- Get

$$\pi_{t+1} = \bar{\pi}^{1-\alpha} (\pi_{t+1})^{\alpha} \Longrightarrow \pi_{t+1} = \bar{\pi}$$

- Success (other than P_0)?

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- Success (other than P_0)?
- How does central bank know P_{t+1} when setting R_{t+1}?

Taylor rule: targeting rule or reaction function?

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- Translation: nice to have, but potentially pie in the sky (at that point, in our world we might as well say $\pi_t = \bar{\pi}$ directly)
- Reaction function: "specifies the central bank's instrument as a function of predetermined endogenous or exogenous variables observable to the central bank at the time that it sets the instrument."