Adding Long-Term Debt

Marco Bassetto (based on Cochrane, 2001)

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Motivation

- Last time, we saw that fiscal news create a jump in the price level
- Cochrane (2005) likens gov't debt to Microsoft stock
  - Microsoft stock is a claim to Microsoft profits
  - Gov't debt is a claim to gov't primary surpluses
- Problem:
  - The price of Microsoft share jumps from one day to the next, very volatile
  - Inflation very sluggish (yes, even now)

One Way of Smoothing Jumps: Long-Term Debt

- So far, all of the debt had one-period maturity
- In practice, government issues many different maturities
- What happens in response to fiscal news in this case?

Revisiting the One-Time Fiscal Shock with Two-Period Debt

- Same economy as in our previous classes
- Now, two government bonds: one-period bonds as before, and two-period bonds $D_{2,t}$ promises to pay $D_{2,t}$ dollars in $t+2$
- Two-period interest rate $R_{2,t}$.

Household flow budget constraint

$$B_{t-1} + M_{t-1} + \frac{D_{2,t-1}}{1+R_t} + P_{t-1}(y - c_{1\,t-1} - c_{2\,t-1}) + A_t - T_t \geq \frac{B_t}{1+R_t} + M_t + E_t(z_{t+1}A_{t+1}) + \frac{D_{2,t}}{1+R_{2,t}} \tag{1}$$

- Used no-arbitrage condition to observe that the price of two-period bonds after one period is $1/(1+R_t)$
- To save notation, lump all bonds maturing in one period in B_t, regardless of when they were issued
- So, B_{t-1} contains one-period bonds issued in $t-1$ and two-period bonds issued in $t-2$
- New definition of nominal wealth

$$W_t := B_{t-1} + M_{t-1} + \frac{D_{2,t-1}}{1+R_t} + P_{t-1}(y - c_{1\,t-1} - c_{2\,t-1}) + A_t - T_t$$

No-Ponzi condition

$$W_t \geq -\limsup_{n\to\infty} \sum_{s=t}^{n} E_t[z_{t,s+1}(P_s y_s - T_{s+1})]$$

with the new definition of nominal wealth

Government budget constraint

$$B_{t-1}^S + M_{t-1}^S + \frac{D_{2,t-1}^S}{1+R_t} - T_t = \frac{B_t^S}{1+R_t} + M_t^S + \frac{D_{2,t}^S}{1+R_{2,t}}$$

$D_{2,t}^S$: Two-period bonds supplied by government

Competitive Equilibrium

Homework for you

New first-order condition

$$\frac{\lambda_t}{1+R_{2,t}} = \beta E_t \frac{\lambda_{t+1}}{1+R_{t+1}}, \quad t \geq 0$$

Note: the transversality condition is unchanged (except for the definition of W_t)

Key Characterizing Equations

- Friedman distortion:

$$u'(c_{1t}) = 1 + R_t, \quad t \geq 0 \tag{2}$$

$$R_t > 0 \Longrightarrow M_t = P_t c_{1t}, \quad t \geq 0 \tag{3}$$

- Fisher equation

$$1 = E_t \left[\beta (1 + R_{t+1}) \frac{P_t}{P_{t+1}} \right] \quad t \geq 0 \tag{4}$$

- Two-period bond pricing

$$\frac{1}{1 + R_{2,t}} = \frac{\beta}{1 + R_t} E_t \left[\frac{P_t}{P_{t+1}} \right]$$

Household PVBC

$$W_0 \geq \frac{R_0}{1+R_0} M_0 + \sum_{s=0}^{\infty} E_0 \left[z_{0,s+1} \left(T_{s+1} - P_s y_s \right) \right]$$
$$+ \sum_{s=0}^{\infty} E_0 \left[z_{0,s+1} \left(P_s (c_{1s} + c_{2s}) + \frac{R_{s+1}}{1+R_{s+1}} M_{s+1} \right) \right]$$

Homework: verify that the above is still correct (with the new definition of W_0)

Revisiting the effect of uncertainty and fiscal news

- Same shock as before
- Introduce uncertainty in a single period, $T_{T+1} = P_T(\bar{T} + \tilde{T}_{T+1})$
- $\tilde{T}_{T+1}$ revealed at time $t < T + 1$, and $E_s \tilde{T}_{T+1} = 0$ for $s < t$

The boring periods

- We still get

$$W_0 = \frac{R_0}{1+R_0} M_0^S + \sum_{s=0}^{\infty} E_0 \left[z_{0,t+1} \left(T_{t+1} + \frac{R_{t+1}}{1+R_{t+1}} M_{t+1}^S \right) \right]$$

- Now household initial wealth includes $D_{2,-1}/(1+R_0)$
- Homework: repeat the analysis from the one-period economy and show that nothing changes in periods $s < t$ and in period $s > t$

A Neutrality result

$$W_t = \frac{P_t}{(1-\beta)(1+\bar{R})}[\bar{c}\bar{R} + \bar{T}] + \frac{\beta^{T-t}\tilde{T}_{T+1}P_t}{1+\bar{R}}$$

Reminder:

$$W_t := B_{t-1} + M_{t-1} + \frac{D_{2,t-1}}{1+R_t} + P_{t-1}(y - c_{1\,t-1} - c_{2\,t-1}) + A_t - T_t$$

- Same equation as with one-period debt
- W_t includes now two-period bonds
- But their value is $D_{2,t-1}/(1+\bar{R})$, predetermined, W_t still known at $t-1$ and cannot respond to $\tilde{T}_{T+1}$

Is Long-Term Debt Irrelevant Then?

- NO!
- Things are different if we play with interest rates (so $R_s \neq \bar{R}$ all the time)
- To simplify life, assume $u(c_{1t}) = \alpha \hat{u}(c_{1t})$ with $\alpha \to 0$ (cashless limit)
- Can abstract from seigniorage revenues
- PVBC (+equilibrium!!) simplifies to

$$W_s = \sum_{v=s}^{\infty} E_s \left[z_{s,v+1} T_{v+1} \right]$$

What if interest rates move around?

Solve again for a generic path $\{R_s\}_{s=0}^{\infty}$ (where R_s can respond to information available at s)

- First, for the one-period-debt economy
- Then, the two-period-debt economy

Periods $s < t$

- PVBC simplifies to

$$W_s = \frac{\bar{T} P_s}{(1+R_s)(1-\beta)}$$

- With one-period debt, W_s predetermined:

$$W_s = M_{s-1} + B_{s-1} - \bar{T} P_{s-1}$$

- Get

$$\frac{W_s(1+R_s)}{P_s} = \frac{\bar{T}}{1-\beta}$$

- Use Euler ($s > 0$)

$$\frac{W_s}{P_{s-1}} = \frac{\beta \bar{T}}{1-\beta}$$

What happens if I move $1 + R_s$?

- P_s goes up proportionally
- Also (for $s < t - 1$)

$$W_{s+1} = W_s(1 + \bar{R}_s) - P_s \bar{T} = \frac{P_s \bar{T} \beta}{1 - \beta}$$

- So future nominal wealth goes up proportionally
- Fisher equation: higher rates, more inflation, nothing on the real front
- Same holds also for W_{t+1} (goes up proportionally); homework

What happens with two-period debt?

$$W_s = M_{s-1} + B_{s-1} + \frac{D_{2,s-1}}{1+R_s} - \bar{T} P_{s-1}$$

- No longer predetermined!!
- Can reduce household wealth in period s by increasing R_s
- Expected changes do not work:

$$\frac{1}{1+R_{2,s-1}} = \frac{\beta}{1+R_{s-1}} E_{s-1}\left[\frac{P_{s-1}}{P_s}\right]$$

$$1 = E_{s-1}\left[\beta(1+R_s)\frac{P_{s-1}}{P_s}\right] \quad t \geq 0$$

- But can make R_t conditional on $\tilde{T}_{T+1}$

Period t

Have

$$W_t = W_{t-1}(1+R_{t-1}) - \bar{T}P_{t-1} + D_{2,t-1}\left[\frac{1}{1+R_t} - \frac{1+R_{t-1}}{1+R_{2,t-1}}\right]$$

$$- \frac{R_{t-1}}{1+R_{t-1}}M_{t-1}$$

$$\approx W_{t-1}(1+R_{t-1}) - \bar{T}P_{t-1} + D_{2,t-1}\left[\frac{1}{1+R_t} - \frac{1+R_{t-1}}{1+R_{2,t}}\right] =$$

$$\frac{\beta \bar{T}P_{t-1}}{1-\beta} + D_{2,t-1}\left[\frac{1}{1+R_t} - \frac{1+R_{t-1}}{1+R_{2,t-1}}\right] =$$

$$\frac{\beta \bar{T}P_{t-1}}{1-\beta} + D_{2,t-1}\left[\frac{1}{1+R_t} - \beta E_{t-1}\left(\frac{P_{t-1}}{P_t}\right)\right]$$

Inflation

Looking forward:

$$\frac{\beta \bar{T} P_{t-1}}{1-\beta} + D_{2,t-1}\left[\frac{1}{1+R_t} - \beta E_{t-1}\left(\frac{P_{t-1}}{P_t}\right)\right] = \frac{P_t}{(1-\beta)(1+R_t)}\bar{T} + \frac{\beta^{T-t}\tilde{T}_{T+1}P_t}{1+R_t}$$

- If R_t is known at $t-1$, same as before (LHS simplifies)
- If R_t covaries negatively with $\tilde{T}_{T+1}$, LHS $\uparrow$ when $\tilde{T}_{T+1}$ goes up...
- ... less need for P_t to adjust
- $\Longrightarrow$ Can get less of a jump in P_t for a given fiscal shock
- Trade-off between inflation and interest-rate smoothing

Why 2-period debt is special in a CIA model

- With two-period debt,

$$W_t = B_{t-1} + \frac{D_{2,t-1}}{1+R_t} - \bar{T}P_{t-1}$$
$$= \frac{\beta \bar{T} P_{t-1}}{1-\beta} + D_{2,t-1}\left[\frac{1}{1+R_t} - \beta E_{t-1}\left(\frac{P_{t-1}}{P_t}\right)\right]$$

- Only R_t affects W_t
- Note: surprises in R_t do not matter for expected inflation

$$1 = \beta E_{t-1}\left[\frac{P_{t-1}(1+R_t)}{P_t}\right]$$

- R_{t+1}, R_{t+2},... irrelevant

N-period debt

- With N-period debt,

$$W_t = B_{t-1} + \sum_{j=2}^{N} \frac{D_{j,t-1}}{1 + R_{j-1,t}} - \bar{T} P_{t-1}$$

- Now, expectations about future interest rates affect the long-term rates
- With N-period debt, R_t, ..., $E_t R_{t+N-1}$ matter $\Longrightarrow$ more smoothing
- ... but the Euler equation tells me that changing $E_t R_{t+j}$ changes future expected inflation
- Trade-off between smoothing current and future inflation

Loglinearization

- Previous expressions nonlinear, messy
- Want to have better intuition, and also run some policy experiments more easily
- Loglinearize around $\pi_t = \bar{\pi}$, constant real debt, geometric maturity structure, $T_t/P_{t-1} = \bar{T}$

Geometric maturity structure: notation

- Face value of debt issued in period $t-1$ maturing in s periods: $D_{s,t-1}$
- Note: $D_{1,t-1} = B_{t-1}$
- Assume $D_{n,t-1} = \phi^{n-1} D_{1,t-1}$
- Value of total debt at the beginning of period t:

$$B_{t-1} \sum_{s=t}^{\infty} \frac{\phi^{s-t}}{1 + R_{s-t,t}}$$

- Definitions $R_{0,t} := 0$ $R_{1,t} := R_t$

Reference steady state

- Euler equation: $1 + \bar{R} = \bar{\pi}/\beta$
- Asset-pricing kernel: $\bar{z} = \beta/\bar{\pi}$
- n-period interest rate (Euler equation for n-period bonds): $1 + \bar{R}_n = (\bar{\pi}/\beta)^n$
- Define $\hat{\phi} := \beta\phi/\bar{\pi}$ (measure of real geometric decay of debt)
- Government present-value relationship:

$$\frac{\bar{T}}{1-\beta} = \frac{\bar{b}}{1-\hat{\phi}}$$

Loglinearization

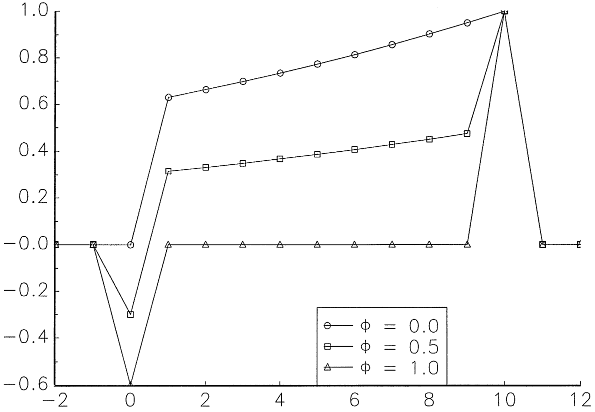
- $E_t[\tilde{R}_{t+1} - \tilde{\pi}_{t+1}] = 0$
- Note: $\tilde{R}_{t+1}$ log-deviation of $1 + R_{t+1}$
- $$\tilde{R}_{n,t} = \tilde{R}_t + E_t \sum_{j=1}^{n-1} \tilde{\pi}_{t+j} = \tilde{R}_t + E_t \sum_{j=1}^{n-1} \tilde{R}_{t+j}$$
- $$(1-\beta)\left[\tilde{T}_t + \beta E_t \sum_{s=t}^{\infty} \beta^{s-t} \tilde{T}_{t+s+1}\right] + \beta(\tilde{\pi}_t - \tilde{R}_t)$$
  $$= \tilde{b}_{t-1} - \hat{\phi}\tilde{R}_t - (1-\hat{\phi}) \sum_{j=1}^{\infty} \hat{\phi}^j E_t \tilde{\pi}_{t+j}$$
- $\tilde{b}_t$: log-deviation of (B_t/P_t) (real one-period debt)

Key equation in innovation form

$$\beta(\tilde{\pi}_{t+1} - \tilde{R}_{t+1}) + (1-\beta)\Big[\tilde{T}_{t+1} - E_t \tilde{T}_{t+1}$$
$$+ \beta \left(E_{t+1} \sum_{s=t+1}^{\infty} \beta^{s-t-1} \tilde{T}_{t+s+2} - E_t \sum_{s=t+1}^{\infty} \beta^{s-t-1} \tilde{T}_{t+s+2} \right) \Big]$$
$$= -\hat{\phi}(\tilde{R}_{t+1} - E_t \tilde{R}_{t+1}) - (1-\hat{\phi}) \sum_{j=1}^{\infty} \hat{\phi}^j \left(E_{t+1} \tilde{\pi}_{t+j+1} - E_t \tilde{\pi}_{t+j+1} \right)$$

- The higher $\hat{\phi}$, the more fiscal shocks can be absorbed by innovations in future inflation rather than current inflation

Debt policy vs. interest-rate policy



Effect on the price level of an increase in 10 period debt at time 0 that is allowed to mature, starting in a steady state with a geometric maturity structure, from Cochrane (2001)

How do we interpret Cochrane's experiment?

(Neglect money)

- Think of a world with only short-term debt

$$B_{t-1} - T_t = \frac{B_t + \Delta B_t}{1 + R_t}$$

- $\implies$ Need $1 + R_t$ to go up one for one with the increase in debt
- Note: we may have $W_t \neq \frac{\beta \bar{T} P_{t-1}}{1-\beta}$ if the policy is a surprise
- Period $t + 1$

$$W_{t+1} = B_t + \Delta B_t = \frac{\beta \bar{T} P_t}{1 - \beta} = \frac{B_{t+1}}{1 + R_{t+1}}$$

- $\implies$ Euler equation means P_t goes up one for one with $1 + R_t$
- $\implies$ works well for the first two equalities
- $\implies$ Need $1 + R_{t+1}$ to go down one for one for last equality

Period $t+2$: back to same as before

$$W_{t+2} = B_{t+1} = \frac{\beta \bar{T} P_{t+1}}{1-\beta}$$

- P_{t+1} unaffected (inflation down one for one, matching interest rate)
- Experiment is best understood as a change in interest rates R_t and R_{t+1}
- Similar intuition for longer-term debt, but now the interest rate changes span N periods and are more complicated

Some deeper questions

- Cochrane lets face value of bonds adjust, interest rate endogenous
- If we set interest rate, we need to let one-period bonds adjust as a residual:
  - Households free to trade money for bonds at given nominal rate
- Tricky to impose geometric maturity structure: how is the supply of other types of bonds determined?
- One possibility: auction long-term bonds in fixed quantities after short-term bonds have been issued

Why are these questions important?

- In section 6.2, Cochrane relates innovations in $D_{n,t}$ to (time t) innovations in current and expected future inflation
- In my structure, those are not the relevant metric
- It's innovations in current and expected future interest rates that matter
- The maturity structure has implications for the way we think about the trade-off between current and future movements in interest rates
- In Section 6.3, Cochrane adds the flexibility that allows him to get to our results (that long-term debt is good)
  - Solve for optimal path of prices
  - Back out quantities of debt from his way of thinking about the problem

Optimal policy experiments

- Cochrane uses variance of inflation, or price level
- In our environment, unexpected inflation is costless
- High interest rates (and volatile interest rates) are bad instead
- By this metric, just fix R_t (close to zero) and let P_t do all the work
- Maturity structure irrelevant

What if we care about the variance of inflation?

$$\beta(\tilde{\pi}_{t+1} - \tilde{R}_{t+1}) + \epsilon_{t+1}$$
$$= -\hat{\phi}(\tilde{R}_{t+1} - E_t\tilde{R}_{t+1}) - (1-\hat{\phi})\sum_{j=1}^{\infty}\hat{\phi}^j\left(E_{t+1}\tilde{\pi}_{t+j+1} - E_t\tilde{\pi}_{t+j+1}\right)$$

- ϵ_{t+1}: innovation in PV of surpluses
- Let $\tilde{R}_{t+1}$ do all the work (only in the cashless limit!)

Shut down R_{t+1}, what if we care about the variance of inflation?

$$\tilde{\pi}_{t+1} - \tilde{R}_{t+1} + \epsilon_{t+1}$$
$$= -\hat{\phi}(\tilde{R}_{t+1} - E_t \tilde{R}_{t+1}) - (1 - \hat{\phi}) \sum_{j=1}^{\infty} \hat{\phi}^j \left(E_{t+1} \tilde{\pi}_{t+j+1} - E_t \tilde{\pi}_{t+j+1} \right)$$

- We want to spread the pain as much as possible
- Long-term debt is good: can spread the effect of the shock across more periods more effectively
- In a real context, related to work by Lustig, Sleet, and Yeltekin (JME, 2008)
- Also related to work by Bhandari, Evans, Golosov, and Sargent