Beauty Contests and Transparency

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Motivation

- We study economies where information is dispersed across households
- There are complementarities across the decisions of different households
- We are interested in the following questions:
  - How do households respond to different information sources?
  - Is information aggregated efficiently?
  - Should policymakers reveal information? "Transparency"

The abstract problem

- Fundamental: θ
- Continuum of identical households
- Households care about tracking θ, but also tracking (or not tracking) each other
- Preferences:

$$u_i(\bar{a}, \theta, \sigma_a^2) = -(1-r)(a_i - \theta)^2 - r(\bar{a} - a_i)^2 + r\sigma_a^2$$

- $r < 1$
- a_i: action of household i
- $\bar{a}$: average action across people
- σ_a^2: variance of actions across people (pure externality)

Complementarities

$$-(1-r)(a_i-\theta)^2 - r(\bar{a}-a_i)^2 + r\sigma_a^2$$

- $r = 0$: single-agent decision problem, no interdependence
- $r > 0$: strategic complementarity, want to choose same action as others
- $r < 0$: strategic substitutability, want to choose action different from others (but still track fundamentals, trade-off)

Distributional assumptions

- Will work with normal distributions
- Uninformative prior on θ: infinite variance (0 precision)
- Informative (common) prior same as having an extra public signal

(Symmetric) Equilibrium: preliminaries

- A strategy profile mapping information into actions
- Given the information, and given that others follow the same strategy, the action implied by the strategy profile is optimal
- Optimality condition:

$$a_i = (1 - r)E_i\theta + rE_i\bar{a}$$

- E_i: expectation based on household i information

Forecasting the forecasts of others

- Start from optimality

$$a_i = (1 - r)E_i\theta + rE_i\bar{a}$$

- Substitute the strategy of others

$$a_i = (1 - r)E_i\theta + rE_i[(1 - r)E_j\theta + rE_j\bar{a}]$$

- Need notation for higher-order beliefs. Define
- Second-order belief

$$E_i^{(2)}\theta := E_i(E_j\theta)$$

- Third-order belief

$$E_i^{(3)}\theta := E_i[E_j(E_k\theta)]$$

- and so on...

Solution with higher-order beliefs

- Iterative substitution

$$a_i = (1 - r)[E_i\theta + \sum_{n=1}^{\infty} r^n E_i^{(n)} \theta]$$

- Infinite regress
- This is a special, simple case: can express as beliefs about exogenous parameter
- In general, we cannot get proof of uniqueness + brute force solution
- $\implies$ for other cases, need guess-and-verify, typically restricting to linear equilibria

Common information benchmark

- Public signal y, $y|\theta \sim N(\theta, 1/\alpha)$
- $\implies$ Posterior distribution $\theta|y \sim N(y, 1/\alpha)$
- $E_i\theta = E(\theta|y) = y$
- $E_i^{(n)}\theta = y$
- Law of iterated expectations holds because information is common
- Equilibrium: $a = y$

Efficiency in the common information benchmark

- Efficient
- Equilibrium ex ante payoff is

$$E_i\left[-(1-r)(y-\theta)^2\right] = -\frac{1-r}{\alpha}$$

- Extra information is always good, increases precision α

Introducing private information

- New signal x_i, $x_i|\theta \sim N(\theta, 1/\beta)$
- LLN applies so that empirical distribution of x_i in the population is also $N(\theta, 1/\beta)$
- ("iid" assumption)
- x_i independent of y (conditional on θ)

Computing expectations

- Order 1:

$$E_i\theta = \frac{\alpha y + \beta x_i}{\alpha + \beta}$$

- Order 2: $E_i x_j = E_i\theta$ so

$$E_i^{(2)}\theta = \frac{\alpha y + \beta\left[\frac{\alpha y + \beta x_i}{\alpha + \beta}\right]}{\alpha + \beta} = \frac{\alpha(\alpha + 2\beta)y + \beta^2 x_i}{(\alpha + \beta)^2}$$

$$= \frac{[(\alpha + \beta)^2 - \beta^2]y + \beta^2 x_i}{(\alpha + \beta)^2}$$

Weight of y

- Order n belief:

$$E_i^{(n)}\theta = \frac{[(\alpha+\beta)^n - \beta^n]y + \beta^n x_i}{(\alpha+\beta)^n}$$

- Higher-order beliefs put more and more weight on public information, less on private
- y is observed by everybody, more useful to forecast what others know

Equilibrium with public and private information

- Can be solved by brute force with higher-order beliefs, or guess that a is linear in y and x_i, compute fixed point of

$$a_i = (1 - r)E_i\theta + rE_i\bar{a}$$

- Solution:

$$a_i = \frac{\alpha y + \beta(1 - r)x_i}{\alpha + \beta(1 - r)}$$

- $r = 0 \implies$ no strategic interaction, single-agent problem, higher-order beliefs irrelevant
- $r > 0$: actions skewed to public information (desire to coordinate)
- $r < 0$: actions skewed to private information (desire to differentiate)

Externalities

$$-(1-r)(a_i-\theta)^2 - r(\bar{a}-a_i)^2 + r\sigma_a^2$$

- Effect of $\bar{a}$: ambiguous:
  - Want it to be predictable, so it can be better tracked by my own action
  - Want it to track θ better, so my action can be close to both θ and $\bar{a}$
- Effect of σ_a^2 clear cut: planner only cares about tracking θ, so complementarities in social welfare

Social welfare

$$W = E[-(1-r)(a_i - \theta)^2]$$

E: expectation ex ante, before receiving signals

$$a_i - \theta = \frac{\alpha(y-\theta) + \beta(1-r)(x_i - \theta)}{\alpha + \beta(1-r)}$$

Get

$$W = -\frac{(1-r)[\alpha + \beta(1-r)^2]}{[\alpha + \beta(1-r)]^2}$$

Comparative statics wrt public information

$$\frac{\partial W}{\partial \alpha} = \frac{1-r}{[\alpha + \beta(1-r)]^3}[\alpha - \beta(1-r)(2r-1)]$$

- $r \leq 1/2 \implies$ more information always good
- $$r > 1/2 \implies \left(\frac{\partial W}{\partial \alpha} > 0 \iff \frac{\alpha}{\beta} > (1-r)(2r-1)\right)$$
- For $r > 1/2$, welfare decreasing in α for α small and increasing for α big
- Optimal transparency: corner solution, either $\alpha = 0$ or $\alpha = \alpha_{\text{max}}$

More on optimal transparency for $r > 1/2$

- Welfare evaluated at $\alpha = 0$:

$$W|_{\alpha=0} = -\frac{1-r}{\beta}$$

- Welfare evaluated at $\alpha = \infty$: 0
- $\implies$ There exists a unique $\alpha^* > 0$ such that $W|_{\alpha=\alpha^*} = W|_{\alpha=0}$

When are public signals beneficial? ($r > 1/2$)

- $\alpha^* = \beta(2r - 1)$
- The higher β, the more precise the public signal has to be to improve welfare

Comparative statics wrt private information

$$\frac{\partial W}{\partial \beta} = \frac{(1-r)^2}{[\alpha + \beta(1-r)]^3}[(1+r)\alpha + \beta(1-r)^2]$$

- $r \geq -1 \implies$ more private information is good
- $r < -1 \implies$ more private information is good if

$$\frac{\beta}{\alpha} > -\frac{1+r}{1-r}$$

- Optimal amount of private information either 0 or the max
- Privately, you always want to use more information

When is private info socially desirable for $r < -1$

- Welfare evaluated at $\beta = 0$:

$$W|_{\beta=0} = -\frac{1-r}{\alpha}$$

- Welfare evaluated at $\beta = \infty$: 0
- $\implies$ There exists a unique $\beta^* > 0$ such that $W|_{\beta=\beta^*} = W|_{\beta=0}$
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$$\beta^* = -\alpha\frac{1+r}{1-r}$$

- The more precise public info is, the more precise private info has to be to be socially desirable

Conclusion

- In the presence of complementarities, public information generates externalities
- More information need not be good, it may generate undesired "herd behavior."