Monetary and Fiscal Policy Rules and their Interaction

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Chapter 1

Time Consistency and Monetary Policy: a Model Based on the Phillips Curve

1.1 Introduction

This book will study the interaction between the "government" and the private sector of the economy. Most of the time, by "government" we will mean jointly the authorities setting fiscal and monetary policy. As we will see throughout this book (but not in this first chapter), the two policies are deeply intertwined, and it is sometimes very tricky to define where monetary policy ends and fiscal policy begins.

There is an important difference between the government and the private sector in the models that we study. The private sector is represented by a large group of agents (in the model, a continuous infinity of agents), so that the actions of a single individual do not affect any of the macroeconomic aggregates. I will call individual agents of the private sector "households," although in richer models this would include firms as well.

In making its decisions, a household takes into account that its actions will have consequences for its payoff, but they have no discernible effect on anybody else's payoffs or information. In other words, a household does not expect a reaction from either the government or other households to its *individual* actions. It is important to understand that the *collective* actions of the households do have an effect on each individual household's payoff, as well as on the government's, and so they will, in general, trigger reactions.

We illustrate this with an example. Consider my choice on where and when to buy a sandwich for lunch. In making this decision, I will consider where I can find sandwiches that I like, how expensive the sandwich is, and how long the line is going to be. However, there is no need for me to think about how my decision affects the lines (or the prices): my individual action will have (almost) no effect on anybody else. Of course, if we *all* choose to go to the same place at the same time, this *does* have an effect on the length of the line, and may trigger a reaction, either by some of us or by the sandwich place (they may open a second location, hire more staff, etc.).

The big difference with the government is that the government is a *large player*. Its individual actions have a direct effect on payoffs and macroeconomic outcomes. For this reason, when moving, the government needs to take into account that its actions may cause reactions by the private sector.[1] This will require us to blend some tools of game theory into our macroeconomic models.

In this chapter's application, due to Kydland and Prescott (1977) and Barro and Gordon (1983), the "government" is best thought of a central bank (CB) that has direct control over inflation. This is the workhorse model to describe the inflationary bias in CB behavior.[2] We will use it to understand the notion of **time inconsistency** and its implications, and the role of private-sector expectations in generating a strategic interaction.

1.2 Barro and Gordon (1983): Setup

We consider here a simplified version of Barro and Gordon's original environment.

The economy evolves over an infinite number of periods $t = 0, 1, \dots$

[1] You may think that some firms are so large that they could have a direct macroeconomic effect. In this case, similar tools could be used to analyze their decision process. This is something that we ignore here.

[2] Of course, these days people are not as worried about *inflation* as they are about *deflation*. There are models that feature different sources of time inconsistency where a deflation bias emerges, but they are more complicated, which is why we will use this one to understand time inconsistency. Also, this model accounts well for the trade-off that central banks perceive in their choice of inflation vs. output stimulation.

In future chapters we will study microfounded models, where the consumption/saving decisions of individual households will be explicitly introduced, albeit in a highly stylized way. Here, the "actions" of the households are simply represented by their choice of an inflation expectation π_t^e at the beginning of period t. In models with full microfoundations, these expectations would enter into long-term pricing decisions of firms, contractual labor arrangements, and that's the way they would then feed into unemployment.

After inflation expectations are locked in, the CB sets actual inflation for the period: π_t.

The interaction of expected and realized inflation determines the unemployment rate, through a relation known as a Phillips curve. Specifically, we assume that there exists an exogenous "natural" rate of unemployment (U_t^n) that prevails if private inflation expectations are correct. This rate is determined by forces that are outside of our model (such as labor-market laws and institutions).

The realized rate of unemployment is determined by the combination of the natural rate of unemployment with *unexpected* inflation:

$$U_t = U_t^n - \alpha(\pi_t - \pi_t^e) \tag{1.1}$$

Unexpected inflation lowers unemployment. In models with explicit microfoundations, this may happen for instance because it lowers real wages (if nominal wages are locked in by negotiations based on previous expectations) and therefore it stimulates labor demand by firms.

In setting its policy, the central bank's goal is to minimize the present-value of its losses:

$$\sum_{t=0}^{\infty} \beta^t [(U_t - kU_t^n)^2 + b\pi_t^2]. \tag{1.2}$$

In equation (1.2), β is a discount factor, that represents the fact that losses far into the future are perceived as less costly than the same losses suffered immediately. In each period, the CB losses are determined by a quadratic penalty in how far inflation and unemployment deviate from some target. The parameter b represents the relative cost of inflation deviations compared to unemployment deviations. We assume here that the CB inflation target is zero.[3] The unemployment target is kU_t^n, where k is a parameter that determines how far the CB target is from

[3] This can be relaxed very easily.

the natural rate of unemployment. When $k = 1$, the CB target coincides with the natural rate of unemployment. When $k < 1$, the CB would like to drive unemployment lower than its natural rate. This is the case in which we are interested.

We are now ready to define an *equilibrium*.[4]

Definition 1 A **Definition: Rational expectations equilibrium where expectations are independent of the past (NE)** is a sequence $\{U_t, \pi_t, \pi_t^e\}_{t=0}^{\infty}$ such that:

1. In each period t, *given* π_t^e and U_t^n, the CB minimizes (1.2) subject to (1.1) by choice of π_t, U_t; and

2. In each period t, $\pi_t^e = \pi_t$.

We discuss here all the elements of NE. Notice that an equilibrium contains a complete description of the evolution of all endogenous variables of our model: unemployment, inflation, and inflation expectations. Second, the qualifier that "expectations are independent of the past" is important. The equilibrium contains a description of the future evolution of the economy *even when the CB chooses actions that are not part of the equilibrium*: specifically, in the equilibrium that we consider, a deviation by the CB in period t carries immediate consequences on inflation and (through the Phillips curve (1.1)) unemployment, but it has no consequences for the future. Without this qualifier, we would need to specify how unemployment, inflation, and inflation expectations depend on the entire history of play.[5] Why do we need to know how the economy will evolve in the future? If the actions of the central bank at time t have future repercussions, then its choice of π_t will take these into account. It is only because we rule out these repercussions that the CB takes into account just the time-t trade-off between inflation and unemployment.

The second equilibrium requirement embeds the notion of rational expectations. Under rational expectations, the households fully understand the model and the CB motives for action, and thus correctly anticipate equilibrium inflation. Rational expectations imply that the CB

[4] We use NE as a short-hand for this equilibrium because there is a connection to the Nash equilibrium of the static stage game that arises in an economy that lives for a single period.

[5] For those of you that have taken game theory, you would then notice an analogy with the definition of a subgame-perfect equilibrium for extensive-form games.

can never fool the households. This clearly a strong assumption, and it can be relaxed in many ways. However, it is (mathematically) the simplest assumption that captures the fact that the CB cannot fool all people all the time, which is almost certainly true in reality. The economic forces that we will uncover here will emerge under weaker assumptions that embed the simple principle that the CB cannot systematically fool the private sector.

1.3 Computing equilibria

In computing NE, we work *backwards* within each period. First, we start from the stage at which households have set their expectations π_t^e and the CB is setting its policy. The first-order condition for minimizing (1.2) subject to (1.1) is

$$-2\alpha[U_t^n(1-k)-\alpha(\pi_t-\pi_t^e)]+2b\pi_t=0, \tag{1.3}$$

which we rearrange to obtain the CB best response:

$$\pi_t=\frac{\alpha U_t^n(1-k)+\alpha^2\pi_t^e}{b+\alpha^2}. \tag{1.4}$$

The CB trades off marginal unemployment benefit with marginal inflation cost. The choice of inflation *given household expectations* is higher:

- The lower k is. A low k implies a greater desire to stimulate the economy and reduce unemployment at the expense of higher inflation.

- The higher inflation expectations π_t^e are, but less than one for one. Higher inflation expectations imply, for any given choice of π_t, higher unemployment. Given the quadratic loss, higher unemployment means a higher marginal cost of unemployment, relative to inflation, and thus more incentive to stimulate the economy. At the same time, the government response is less than one for one, because inflation is costly too; if inflation went up one for one, the CB would take all the hit on inflation and none on unemployment (see equation (1.1)), so that the marginal cost of inflation would be too high instead.

- The lower b is. A lower b implies that the CB cares about inflation less, and is thus more willing to use inflation as a tool to stimulate the economy.

Notice that, even though we have rational expectations, the CB treats π_t^e as given. When it's the CB's turn to make a decision, expectations are locked in, and so in principle the CB is free to fool the households. This never happens because the households understand that the CB will make an optimal choice based on (1.4) and set their expectations appropriately. This is the second step in our computation of NE: substitute $\pi_t^e = \pi_t$ into (1.3) or (1.4), which yields

$$\pi_t = \frac{\alpha U_t^n (1-k)}{b}. \tag{1.5}$$

When households set their expectations of inflation based on (1.5), the CB is free ex post to fool them and choose any other value: however, its optimal choice based on (1.4) is instead to precisely choose that same value.[6]

Finally, since $\pi_t = \pi_t^e$, equation (1.1) implies that $U_t = U_t^n$. Ex post, the CB perceives a trade-off between inflation and unemployment. However, ex ante inflation expectations adjust to the CB incentives, and so unemployment is entirely driven by the exogenous natural rate, and is unaffected by CB policy.

Equilibrium inflation is higher:

- The lower k or b are (for the same reasons highlighted when we discussed the CB best response);

- The higher α is. This happens because a high α implies that inflation is more effective at cutting unemployment, and so it increases the temptation to resort to inflation. You should notice here the irony that occurs because of the strategic behavior. If households had purely backward-looking expectations, then a higher α would mean that the CB would need actually less of an inflation surprise to deliver the same unemployment reduction: higher α would be good news. But ex ante the CB cannot really affect unemployment, so α increases the temptation for no gain.

[6] In this problem, this equilibrium value is unique: if households set their expectations to any other value, the CB would have an incentive to choose an inflation level that is different from what the households expect.

What the CB faces is *time inconsistency*: its preferred actions ex ante and ex post are different.

- When anticipated, inflation does *nothing* for unemployment, so the optimal choice is to set it to zero.
- Yet, ex post, once private sector expectations are locked in, the CB perceives a trade-off and gives in.

1.4 Possible solutions to time consistency problems

1.4.1 Commitment

When possible, a solution to time consistency is to commit to actions beforehand, so that ex post temptation is impossible. A famous example of this solution occurs in Homer's Odyssey. Ulysses is about to sail past where the sirens sing. The sirens' chant is supposed to be incredibly beautiful, so beautiful that mariners are known to dive into the sea to try to reach the sirens and drown in the attempt. Ex ante, Ulysses would like to hear the chant without drowning, but he knows that, ex post, his preferences will change and he will want to dive into the sea upon hearing the chant. He solves the quandary by asking his crew to tie him to the boat's mast: this *removes* the option of diving. [His crew wears ear plugs instead].

As in the story about Ulysses and the sirens, commitment is about removing the ability to make choices ex post. This requires some institutional mechanism that makes it *impossible* for the CB to change inflation after private-sector expectations are set. Unfortunately, in practice, it is hard to deny a CB the power to take actions ex post, which is why commitment is not really a solution in practice.

Nonetheless, let us consider what the equilibrium would look like under commitment. Specifically, we now reverse the timing of choices within each period t:[7]

[7] Notice that this is *partial* commitment. Full commitment would require the CB to set all of its actions at time 0. However, for this application, one-period commitment is enough to avoid the undesirable effects of time inconsistency.

- At the beginning of period t, before household form expectations, the CB sets inflation π_t, which it will not be able to revise afterwards;

- After observing π_t, households set their expectations. Rational expectations will then imply $\pi_t^e = \pi_t$, for whatever value the CB set.

Definition 2 A **rational expectations equilibrium with one-period commitment** is a sequence $\{U_t, \pi_t, \pi_t^e\}_{t=0}^{\infty}$ such that:

1. In each period t, $\pi_t^e = \pi_t$; and

2. In each period t, the CB minimizes (1.2) subject to (1.1) by choice of π_t, U_t, *taking into account that* $\pi_t^e = \pi_t$.

To compute this equilibrium, we work backwards once more. Now it is households that move last and set $\pi_t^e = \pi_t$. When it is the CB's turn to move, thus, it minimizes its loss function subject to (1.1), but it takes into account that π_t^e will subsequently be set equal to π_t and thus makes this substitution into (1.1). With this substitution, (1.1) yields

$$U_t = U_t^n :$$

there is no trade-off between inflation and unemployment, so the optimal choice is to set inflation at its target (zero) and let unemployment be at the natural rate, where it would be anyway. Under commitment, unemployment same as under discretion (i.e., without commitment), but there is no inflation, so the CB is better off!

1.4.2 Delegation

An alternative solution has been proposed by Rogoff (1985). We noticed that equilibrium inflation from (1.5) is lower, the higher b is. Suppose now that there are many different people, with different preferences for b, and that the choice can be delegated to one of them ex ante, before expectations are set. Delegation is a form of commitment. A low-b person would want ex post to use inflation to reduce unemployment. However, ex ante, she realizes that it's a futile attempt and it's best to remove this option. By delegating the decision to a high-b individual, who is less subject to temptation, it is thus possible to mitigate the effects of time inconsistency, attaining the same level of unemployment, but with lower inflation. If an election is held ex ante, all types will thus unanimously vote for the highest b type to act as central banker.[8]

1.4.3 Reputation

In our main section, we described equilibria in which private expectations independent of the past. It is often possible to have equilibria with better outcomes when future expectations do depend on the past. As an example, suppose that households trust the CB to set zero inflation if they have always observed zero inflation in the past, but set their expectations to the value given by (1.5) otherwise. With this household strategy,[9] a CB that faces zero inflation expectations might find it optimal to deliver on them. Specifically, by choosing a different value, the CB achieves a one-time gain in the period it fools the households; however, it is then stuck with higher inflation expectations for ever, which brings future costs. When β is sufficiently close to 1, so that the CB is relatively patient, it is definitely desirable for the CB to deliver on zero inflation, forgoing the immediate gains but preserving favorable expectations for the future.

Reputation seems a good way to achieve the commitment outcome without having access to a commitment technology (at least, if the CB is sufficiently patient). However, we do not have a theory of how the CB could coordinate households on this equilibrium rather than the one in which inflation expectations are independent of the past; both are completely valid equilibria of the dynamic game. In fact, there are many other equilibria, where inflation expectations can be complicated functions of the past. There are equilibria in which inflation expectations are different in even and odd periods. There are even equilibria that are *worse* than the equilibrium in which inflation expectations depend on the past. The theory of repeated games gives somewhat

[8]This result would change whenever flexibility is valuable ex post. As an example, if the economy is subject to random shocks, low-b types might want to use inflation variability as a shock absorber, and will thus have to trade off the time-consistency benefits of electing a high-b type with the costs in terms of lost flexibility.

[9]Notice the analogy with a game: this is a strategy, in that it describes what households will do under all possible scenarios of past histories.

weak predictions, so relying purely on reputation is a shaky foundation to ensure good outcomes.

Chapter 2

A Cash-in-Advance Economy

2.1 Introduction

In our first chapter, we assumed that the central bank had direct control over inflation. In practice, central banks are not a price-control board: inflation is determined by their actions on money supply and interest rates, as well as by fiscal policy and the expectations of the private sector (and, at least over short horizons, by other exogenous shocks, such as shocks to oil prices).

In our next chapters, we will study in detail the fundamental forces that determine inflation. Our focus will be on the *long run*. The models that we will employ are too simple to explain why inflation moves around one or two percentage points from one year to the next. Nonetheless, they are extremely helpful in understanding why inflation is low now and was high in the 1970s, why inflation spikes during times of war or when government finances are in trouble, and in analyzing the conditions that must be met for price stability to persist.

In this chapter, we lay out a framework that we will be using for most of the rest of this book. We will use this framework to analyze and better understand a number of different policies. To this end, we will adopt a model that features deeper microfoundations, where the incentives faced by the private sector and the choices faced by the households are more clearly spelled out.

The main way in which central banks conduct monetary policy is through *open-market operations*, whereby they buy or sell bonds (often domestic government bonds, but not always, as we will have a chance to discuss) in exchange for *monetary base*, which can take the form of cash or electronic reserves held by commercial banks (and, in some cases, other authorized entities). While open-market operations are by far the most common way of conducting day-to-day business, they have been used by central banks around the world to achieve several different intermediate targets. In modern times and for most of the developed countries, this intermediate target has been about (short-term) interest rates. Up until 2008, in the U.K., as well as the U.S. and the Eurozone, the interest-rate target was the main piece of news in reporting central bank decisions. Other prominent examples of a target are defending a fixed exchange rate (e.g., Europe before the Eurozone, and many developing countries to this day), or controlling the quantity of money (quantitative easing, money targets in the early 1980s,...).[1]

To properly analyze monetary policy, our model needs thus at least three key ingredients:

- A motive for the existence of money. Up until 2008 (and even now, in most of the world), carrying money implied forgoing the opportunity of earning interest by investing in government bonds. If money offered no additional benefits compared to a bond investment, nobody would carry it.

- Government bonds. As we observed, most of the time, this is what central banks use for open-market operations. Moreover, in times of fiscal stress, when Treasury has trouble placing its bonds, it typically leans on the central bank to buy some of them, with important implications that we want to explore.

- Taxes. There must be a way to repay those government bonds!

The model that we develop here is highly stylized. It misses many elements that are particularly important to explain the day-to-day variation in inflation. Prominent among these elements is a friction that prevents immediate adjustment in prices (e.g., "sticky prices"). You have studied extensively this friction in previous courses. We do not include it here not because it

[1] Of course, these are only intermediate targets, because the final goal is often price stability, with the addition of helping to smooth unemployment and output fluctuations, and, in times of fiscal crises, to assist in financing the needs of the government.

is unimportant, but because it would make our analysis much more difficult without overturning any of our long-run results. Since we are interested in studying what determines inflation in the medium to long run (say, over 5 years or longer), these frictions would represent a distraction.[2]

Another element that is missing from our analysis is heterogeneity within the private sector. This heterogeneity is often important to understand the motives behind policymakers' actions. At medium to high rates, inflation is a tax on the poor, who do not have access to sophisticated financial instruments and hold a large fraction of their assets in cash.[3] Once again, the reason for abstracting from this is that it greatly complicates the analysis but it does not change the conclusions that we will reach about the way fiscal and monetary policy determine inflation: it can be useful to explain why certain policies are chosen and not others, but not as much to understand the effect of those policies on inflation.

2.2 Setup

To describe a model with microfoundations, we need to introduce the following elements:

- The commodity space: which goods are available? What is the time dimension? Is there a spatial dimension to our economy? (Wheat in Chicago is not the same good as wheat in London). Are there different states of nature? (Wheat after a drought is not the same good as wheat in a year of perfect rainfall).

- The agents: who is making decisions? When uncertainty is present, we include a fictitious player, called "nature," that makes random decisions.

- Agent preferences: what drives agents to make certain decisions and not others?

[2]It is worth noting that even state-of-the-art models, that include lots of frictions and realistically match the behavior of many macroeconomic aggregates, struggle explaining high-frequency variation in inflation. A significant fraction of this high-frequency variation ends up being attributed to "mark-up shocks," which is essentially saying that inflation is high because firms feel like charging high prices.

[3]Inflation can be a boon for middle-aged, middle-class households, that have most of their wealth locked in housing and carry a mortgage balance.

- Technology: how are goods produced? Can they be stored? Are there intermediate inputs?
- Markets and information: what can be traded? When? What do people know, and when do they know it?
- An equilibrium concept. This will be a description of how all of the elements above lead to a prediction about what will happen.[4]

2.2.1 The commodity space

Time in our economy will be divided into a discrete and infinite number of periods. 0 will be our first period, and the economy will evolve over infinite periods $t = 0, 1, 2, \ldots$.

Our economy will also feature a spatial dimension: there will be a large number of identical "islands." In fact, we will assume that there is a "continuum" of these islands, so what happens in just one island will not have an effect on any of the aggregates.

We will study economies that feature no uncertainty, so we do not need to worry about different states of nature (sometimes I will briefly mention some results that would be true with uncertainty, without deriving them).

On each island, there will be a single consumption good. The economy will also feature "money," an intrinsically useless object that will be the same across all islands.

2.2.2 The agents

There is a large number (again, a "continuum") of identical households. Here too the continuum assumption means that what an individual household does has no effect on macroeconomic aggregates.

[4]Our equilibrium concept (competitive equilibrium, with a smattering of elements from game theory here and there) will be simple in that it features no monopoly power, markets that clear, and prices that adjust instantly. But many "disequilibrium" models are simply models that have a more complex notion of an equilibrium; to mention but one example, one can define equilibria where sometimes demand exceeds supply and rationing occurs, because some frictions prevent prompt adjustment.

In addition, the economy has a monetary authority, which we will call "central bank" (CB), and a fiscal authority, that we will call "Treasury." Often we will study the CB and Treasury as a single entity, in which case we will call them "government."

2.2.3 Preferences

Household preferences are described by the following utility function:

$$\sum_{t=0}^{\infty} \beta^t [u(c_{1t}) + c_{2t}], \tag{2.1}$$

c_{1t} and c_{2t} is the consumption of two goods from different islands. For reasons that will be clear shortly, we will call type-1 goods "cash goods" and type-2 "credit goods." β is a time discount factor. u is a utility function that is strictly concave and satisfies the following "Inada" conditions:

$$\lim_{c_{1t} \to 0} u'(c_{1t}) = \infty$$

and

$$\lim_{c_{1t} \to y} u'(c_{1t}) = 0.$$

Strict concavity ensures that the household first-order conditions in their maximization problem are necessary and sufficient. The Inada conditions ensure that the solution is interior: households will not want to consume all of the endowment (to be described) in the form of just cash goods or just credit goods. When households have zero consumption of cash goods, the marginal value of an extra unit of cash goods is infinite, and they would give up arbitrarily large amounts of credit goods to avoid this. Conversely, when households consume all of y in the form of cash goods, the marginal value of cash goods is zero, and households would be willing to give up cash goods for arbitrarily small extra amounts of the credit good.

We make our life simple by assuming that the utility function is linear in credit goods. This will make some of our expressions nicer, but the results that we will derive are more general.

We will be silent on what drives government policy. Rather than specifying preferences for the government (as we did in the last lecture), we will simply explore the implications of many different strategies that Treasury and the central bank could pursue.

2.2.4 Technology, markets, and information

Each household has a home island, where it starts each period with y units of the good; the good cannot be stored from one period to the next. In contrast, money is perfectly storable, and can be produced for free by the CB.

At the beginning of each period t, all households travel to a central location, where an asset market is present. Goods cannot be carried to this location: only money and bonds can be traded here. In this location, the Treasury levies taxes in an amount T_t, which are payable in money. The government and households also settle their outstanding debt (in money), and households and the government can buy or sell new debt. To keep things simple, we will assume that all of this debt is risk free (there is no uncertainty in our economy), that it lasts one period (it is sold at t and repaid at $t+1$, although it may be rolled over into new debt), and that it is written in nominal terms (we will call "dollars" the unit of account). The difference between money and debt is that money is a physical object, whereas debt is purely a bookkeeping entry, that is not accessible until the next meeting of the asset market.

Each household has two members: a shopper and a worker. After the asset market closes, shoppers and workers divide. The workers return to their home island and sell their endowment of the good there. They can also buy goods in their home island. Since all the households in the home island know each other, transactions among households on the home island can be done on credit: the seller will recognize the buyer in the asset market next period, and would be able to take her to court if she did not settle her debt at that time. The goods acquired on the home island are the "credit" goods, that enter linearly in the household utility function.

Shoppers travel to foreign islands. Since there is a continuum of those islands, they always travel to a different one. Shoppers are anonymous; the workers of the island that they visit would not be able to recognize them. For this reason, shoppers cannot buy goods from the workers on credit, since workers would not know from whom this credit should be collected. Fortunately, everyone recognizes money, so shoppers can use money instead. The goods that they purchase are "cash" goods, that enter in the utility function through the utility $u()$.

Notice that the same good is a cash good for some people (the shoppers that traveled to the island) and a credit good for others (the workers on their home island).

After trade has taken place, shoppers travel to their home island with the cash goods that they purchased, and workers and shoppers jointly consume what they acquired.

2.2.5 An interpretation of our model

What we described is a toy economy. It is important to understand its relationship with the real world, so we go back to our list of key ingredients and see how the model delivers them. Most of the elaborate description of the model is driven by the need to create a role for money. Here, money is used to settle anonymous transactions that could not be carried out with credit. In the real world, money is said to have a "liquidity" role. This is an ambiguous term that means different things in different contexts, but the key element is that there are transactions that require payment in money. The textbook example is a random person going to the corner store. But these days corner stores often accept credit cards. (Another example is a person buying illegal drugs; drug dealers do not take credit cards). However, there are other examples that are much more important. During the financial crisis, Bear Sterns was unable to get credit even using a short-term U.S. Treasury bill as collateral: they could not settle their debts even with the most liquid and risk-free dollar security. Our abstract economy captures this special role of money that makes some transactions possible that otherwise could not take place.

What "money" is also depends on the context. There are many definitions of money, ranging from very narrow, such as M0 (essentially currency and bank reserves) to much more extensive (such as M3, which includes on-demand deposits as well as many short-term deposits). When we think about liquidity services in general, liquid instruments can include a variety of assets that can be sold at short notice for a highly predictable price.

For us, money will chiefly be M0. We will take a narrow view because M0 is the one that is most connected to government finances.

2.3 Equilibrium

A *Competitive equilibrium* of our economy will include the following elements:

- An allocation $(c_{1t}, c_{2t}, M_t, B_t)_{t=0}^{\infty}$, that describes household consumption of cash and credit goods, money holdings (M_t) , and bond holdings (B_t);
- A price system $(P_t, R_t)_{t=0}^{\infty}$, where P_t is the price of goods in period t and R_t is the one-period nominal interest rate between periods t and $t+1$;
- and a government policy $(T_t, B_t^S, M_t^S)_{t=0}^{\infty}$, where B_t^S is the supply of bonds by the government and M_t^S is the supply of money.

such that:

- Given the price system and government policy (their entire sequence), the allocation maximizes a household utility subject to its constraints. Notice that this requirement embeds rational expectations, because we assume that the household correctly anticipates the entire future sequence of prices and policy when making its time-0 decision.
- Markets clear: this includes the bond market, $B_t = B_t^S$, the money market, $M_t = M_t^S$, and the goods market $c_{1t} + c_{2t} = y$.
- The government budget constraint is met period by period. This requirement is redundant, because it will always be satisfied when the others are.

A couple of considerations are in order:

- Since all households and islands are identical, they will optimally choose the same allocation, and prices will be the same on all islands. In characterizing an equilibrium, we will need to think of the consequences that would befall on each household if it chose something other than the plan followed by all the others; but the continuum assumption means that such a deviating household has no effect on aggregates, and so this deviation would have consequences for one household only; all others would be able to carry out their original plan. This is different from a two-player game, where what I do has an effect on what you can do in turn.

- The equilibrium features a single price at which both cash and credit goods are sold. We could in principle allow for different prices, but we would find that, as part of the equilibrium, the two prices are the same. Why is this the case? Think of a worker that is selling its goods. If she sells to a credit buyer at a price P_t in period t, she will receive P_t dollars in the asset market in period $t+1$, when debts are settled. If she sells to a cash buyer, she receives P_t dollars in money right away. She could use that money to buy goods on the home island, but this carries no advantage over buying goods on credit there. It would be great if she could send the money to the shopper, that needs cash on the foreign island. However, the shopper is gone and will not be back until the end of the period, when it is too late to buy goods. So, the worker will carry the newly acquired money into the asset market in period $t+1$, when those dollars will have exactly the same value as the credit balance accrued from selling on credit. If the price of credit-good transactions and cash transactions is the same, then the worker will be indifferent; otherwise, she would cater to just one group of customers, and prices would have to adjust accordingly.

To characterize competitive equilibria, we need to write down explicitly the constraints faced by households. First, in each period t in the asset market a household faces the following constraint:

$$B_{t-1} + M_{t-1} + P_{t-1}(y - c_{1\,t-1} - c_{2\,t-1}) - T_t = \frac{B_t}{1+R_t} + M_t. \tag{2.2}$$

A household carries from last period a claim B_{t-1} in bonds, plus any money balances that it has not spent in the previous period $(M_{t-1} - P_{t-1}c_{1\,t-1})$. In addition, it carries a balance of $P_{t-1}y$ for the goods it sold in the last period (part of this is a credit and part is money, but the two are equivalent at this stage) and it has to pay taxes in an amount T_t and settle its credit-good purchases $P_t c_{2t}$ (a debit entry). The household divides the resulting balance between money M_t and bonds that promise to pay B_t in period $t+1$ and sell at a discount given by the interest rate.

If this were the only constraint faced by the households, it could satisfy it simply by going into debt (B_{t-1}) and rolling over that debt for ever. We prevent this by assuming a "no-Ponzi" condition that limits how far into debt a household is allowed to go in each period:[5]

$$B_t/P_t \geq -\underline{B},$$

where $\underline{B}$ is best thought of a large number that is there just to make sure that debt is not rolled over for ever.

While $B_t < 0$ is allowed, which is interpreted as a household going into debt, money cannot go negative:

$$M_t \geq 0.$$

If you try to print your own money (the equivalent of going negative), you will find that the justice system will have something to say.

The final constraint for the households is the one that requires that all purchases of cash goods be done in cash, and is known as the "cash in advance" constraint, because the cash must have been acquired in the asset market beforehand:

$$M_t \geq P_t c_{1t} \tag{2.3}$$

The government is subject to a similar period-by-period budget constraint, given by

$$B_{t-1}^S + M_{t-1}^S - T_t = \frac{B_t^S}{1+R_t} + M_t^S. \tag{2.4}$$

All government transactions take place in the asset market.[6] At the beginning of the period, the government has outstanding debt claims $B_{t-1}^S + M_{t-1}^S$: M_{t-1}^S are actual dollars, and B_{t-1}^S are promises to deliver dollars. Households pay their taxes, and this reduces the debt claims by T_t.

[5] If this condition were invented today, it would probably be called a "no-Madoff" condition. In case you are curious about Ponzi's story, here is the Wikipedia entry: `http://en.wikipedia.org/wiki/Charles_Ponzi`. Of course, as with all Wikipedia entries, you might want to double-check the facts if they are important for you, particularly regarding the fine details.

[6] We assumed away government spending, because it does not play a role in our story. We could reintroduce it. If we did, it would not matter whether the government pays for its goods in money or cash.

The government may settle the remaining claims either by delivering money (M_t^S) or by issuing new bonds, promises to deliver money in period $t+1$, that carry an appropriate discount given by the interest rate.

An important question is whether the government has a no-Ponzi condition. We will assume that it does not. The reason is that debt is a promise to deliver money, and (unlike the households) that the government can freely produce money. There is never an issue that the government would be unable to repay. Of course, the *value* of the money will be heavily influenced by *how* the government repays its debt. This is at the heart of much of what we will do. Later on in the course, we will split the government into Treasury and the CB. Treasury cannot print dollars, so it *does* have a transversality condition, unless it can force the CB to print money to repay debts in its stead.

Having described the constraints faced by the households and the government, we are ready to derive the conditions that must be satisfied in a competitive equilibrium. Consider first the household maximization problem. Writing $\beta^t \lambda_t$ for the Lagrange multiplier on the household budget constraint and $\beta^t \mu_t$ for the Lagrange multiplier on the cash-in-advance constraint, the household Lagrangean is

$$\begin{aligned}&\sum_{t=0}^{\infty} \beta^t \Big\{ u(c_{1t}) + c_{2t} + \\ &\lambda_t \Big[B_{t-1} + M_{t-1} + P_{t-1}(y - c_{1\,t-1} - c_{2\,t-1}) - T_t - \frac{B_t}{1+R_t} - M_t \Big] + \\ &\mu_t (M_t - P_t c_{1t}). \Big\}\end{aligned}$$

The first-order conditions are as follows:[7]

- Cash goods:

$$u'(c_{1t}) = (\beta \lambda_{t+1} + \mu_t) P_t, \quad t \geq 0; \tag{2.5}$$

[7]Recall that we assumed that u is strictly concave. You can prove that the constraints determine a (weakly) convex set of admissible allocations. For this reason, the first-order conditions are necessary and sufficient. This problem is harder than the ones that you have seen in most other courses, because the object with respect to which we maximize is infinite dimensional. This brings about some complications, most notably the transversality condition discussed below, but you can mostly forget about these complications and treat the problem above as a regular Lagrangean.

- Credit goods:

$$1 = \beta\lambda_{t+1}P_t, \quad t \geq 0; \tag{2.6}$$

- Money:

$$\lambda_t = \mu_t + \beta\lambda_{t+1}, \quad t \geq 0; \tag{2.7}$$

- Bonds:

$$\frac{\lambda_t}{1+R_t} = \beta\lambda_{t+1}, \quad t \geq 0. \tag{2.8}$$

In addition, there is one more necessary condition, which intuitively is best interpreted as coming from an unspecified multiplier on the no-Ponzi condition. We call this a *transversality condition.*

$$\liminf_{t\to\infty}\left(\frac{B_t}{1+R_t} + M_t\right)\prod_{s=0}^{t-1}(1+R_s)^{-1} = 0 \tag{2.9}$$

In words, this condition states that the value of bonds and money far into the future, discounted in today's dollars, should become very small. When this condition is violated, either a household is running a Ponzi scheme (which we ruled out) or a reverse Ponzi scheme, where it accumulates infinite wealth that it never uses to consume. Accumulating wealth for the sake of it may be appealing to Uncle Scrooge, but not to our households, that only enjoy consumption.

We can get rid of the Lagrange multipliers and obtain two relationships that will be key in much of what we do (and that you would do well to learn by heart).

2.3.1 The Fisher equation

Substitute (2.6) into (2.8):

$$\frac{P_{t+1}}{P_t} = \beta(1+R_{t+1}) \quad t \geq 0. \tag{2.10}$$

This identifies a *positive* relation between interest rates and inflation. At first, this may seem surprising. After all, we think that central banks *raise* rates to *fight* inflation! However, if you look at a longer horizon, this relationship emerges very clearly (and unsurprisingly) in the data. In high-inflation countries, interest rates must be high: inflation erodes the value of credit, and nobody would lend at low rates.

As usual, our model misses the short-run features that may create a short-run incentive to raise rates to fight inflation on a day to day basis, but it instead captures the key long-run relationship that households will have to be compensated for inflation in order to willingly lend to the government (and each other).

2.3.2 The Friedman distortion

Substitute (2.7) into (2.8), then use (2.6):

$$\beta\lambda_{t+1} + \mu_t = \beta\lambda_{t+1}(1 + R_t) = \frac{1 + R_t}{P_t}.$$

Use this expression to get rid of the Lagrange multipliers in (2.5):

$$u'(c_{1t}) = 1 + R_t, \quad t \geq 0. \tag{2.11}$$

Remember that 1 is the marginal utility of credit goods. When $R_t > 0$, equation (2.11) captures what is known as the *Friedman distortion*. Technologically, one unit of credit goods can be turned into one unit of cash goods. Furthermore, while cash goods require money to be purchased, money is free to produce from a societal point of view. Yet, when $R_t > 0$, the marginal utility of cash goods is higher than the marginal utility of credit goods. This is because, by setting $R_t > 0$, the government charges an opportunity cost for holding money (forgone interest), even though money is technologically free.

The incentive to economize on money shows up in comparing the first-order conditions for money and bonds, equations (2.7) and (2.8). In these equations, the Lagrange multipliers λ_t and λ_{t+1} are always strictly positive. Mathematically, one way to see this is to use the first-order condition for credit goods, equation (2.6). On the economic front, the Lagrange multipliers λ are the multipliers on the household budget constraint: if they were not binding, households would want to consume infinite goods (at least infinite credit goods, since there is no question about the cash-in-advance constraint with the credit goods). This is why the first-order condition for credit goods tells us that the multipliers will be positive.

Next, compare (2.7) and (2.8). If $R_t = 0$, then we obtain $\mu_t = 0$. Conversely, if the nominal interest rate $R_t > 0$, then $\mu_t > 0$. μ_t is the Lagrange multiplier on the cash-in-advance constraint.

When $R_t = 0$, households earn the same return (zero in nominal terms) whether they keep their savings in money or bonds; there is thus no need to economize on money, and they are happy to hold cash in excess of what they need for transactions. When $R_t > 0$, bonds are a better saving technology than money. In this case, households will keep money strictly for their transactions needs, so that the cash-in-advance constraint (2.3) will hold as an equality. We thus established the following:

$$R_t > 0 \Longrightarrow M_t = P_t c_{1t}, \quad t \geq 0 \tag{2.12}$$

As a side remark, note that this is the only distortion in our model. In this model, inflation is costly, but it is costly only because it is associated with high nominal interest rates (because of the Fisher equation). High expected inflation leads to a high opportunity cost of money (the nominal interest rate), which in turn leads households to spend less in cash goods and more in credit goods. This cost only applies to *expected* inflation. A surprise jump in the price level would have no effect on welfare. This is very different from the Barro-Gordon model, where expected and unexpected inflation have the same cost for the government (by assumption).

2.4 Equilibria under money supply rules

In order to compute a competitive equilibrium, we need to know more about government policy, i.e., how taxes, interest rates, and the supply of money and bonds are set. This will be the main theme of what we will study in these first weeks. In this section, we start by analyzing *money supply rules*, where the central bank fixes the amount of money available to the private sector.

In a money supply rule, the government sets in each period t the amount of money that is available to the private sector, potentially as a function of the entire history up to that point in time.

A simple example of a money supply rule is a constant money growth rule: the government supplies some amount of money M_0^S in period 0, and lets money grow at the rate q from then on, independently of what happened in the past:

$$M_{t+1}^S = (1+q) M_t^S, \quad t \geq 0. \tag{2.13}$$

In addition to specifying the money supply process, we need to say something about fiscal policy. We will see in later chapters that this choice may be very important for determining inflation. For now, we assume that the government does not issue bonds, so

$$B_t^S = 0. \tag{2.14}$$

This policy may seem very special (and it is), but there are many alternative specifications that would yield the same results, as we will discuss.

With (2.13) and (2.14), the government budget constraint (2.4) tells us what taxes must be:

$$T_t = M_{t-1}^S - M_t^S = -qM_{t-1}^S, \quad t \geq 0,$$

and

$$T_0 = B_{-1} + M_{-1} - M_0^S. \tag{2.15}$$

We treat period 0 differently because we do not discuss how the initial bond and money holdings of the households came to be. The best way to think about this is that at time 0 a completely unexpected reform takes place. The households come into this period with some nominal bonds B_{-1} and M_{-1}, and we study how the economy evolves from there.[8] Since B_{-1} and M_{-1} do not matter independently, often we lump them together and define some initial nominal wealth $W_{-1} = M_{-1} + B_{-1}$ that the households inherit from the past and have at the beginning of period 0.

[8]This may seem suspicious to you: we have a model with rational expectations (even more, perfect foresight!) and yet somehow period 0 feels so special because we forget how expectations were formed in period -1 (and -2, -3,...). In an economy without uncertainty, this feels somewhat strange, and indeed some things will be true of period 0 that will not be true in subsequent periods. However, if we introduced uncertainty, period 0 would not be as special. Suppose that what happens in period 1 depends on the outcome of a random variable that has a continuous distribution, say uniform between 0 and 1. This random variable is only observed in period 1. Each realization of this random variable is a probability zero event, so it has no effect on what people expect as of period 0; only the expectation taking all realizations together will matter and affect the agents' economic decisions. Because prior expectations do not put a constraint to what happens in each realization, it's as if the world started anew from then on, just as it does in period 0 for us. Of course, the expectations would link what can happen across realizations, because across realizations some conditions (a version of the Fisher equation in particular) must be true in expectation.

2.4.1 The logarithmic case

We consider first a case in which a money supply rule is successful at pinning down the price level and inflation. Assume that the money growth rate $q > \beta - 1$: money either grows, or it may shrink, but not at too fast a rate. Suppose further that the utility function for cash goods is $u(c) := \log c$; then, the Friedman distortion (2.11) yields $c_{1t} = (1 + R_t)^{-1}$. Assume for now that the cash-in-advance constraint holds as an equality in period t, and replace the value for c_{1t} that we just found:

$$M_t = \frac{P_t}{1 + R_t}.$$

Next, use the Fisher equation (2.10) to substitute for the interest rate; simplifying, we obtain

$$M_t = \beta P_{t-1}, \quad t > 0 \tag{2.16}$$

In equilibrium, money supply and money demand must be equal, so we know what M_t is and we can derive the price level from equation (2.16):

$$P_t = M^S_{t+1}/\beta = (1 + q)^{t+1} M^S_0/\beta.$$

Success! This equation gives us a unique value for the price level in each period. It also tells us that, if money grows at the constant rate q, the same is true of the price level. This is also a success for monetarism: here, inflation is high if and only if money growth is high.

You may wonder about the role of the assumption $q > \beta - 1$. To understand it, go back to the Fisher equation (2.10) and use the fact that $P_{t+1}/P_t = 1 + q$. We can solve for the interest rate and obtain

$$R_t = (1 + q)/\beta - 1, t > 0.$$

If $q < \beta - 1$, we would obtain a negative nominal interest rate: this is impossible! If the interest rate is -1%, I can borrow $1 today and promise to repay $0.99 next year. I can pocket the dollar in the form of money, which will give me $1 next year: I use $0.99 to repay the loan, and pocket $0.01. I earned money without taking any risk and without investing any money of my own (I only used borrowed money): this is known as an *arbitrage*. At these prices, people would borrow infinite amounts, carry over the balance as money, and make infinite profits in the process, which cannot happen in equilibrium.[9]

The analysis of the case $q = \beta - 1$ is considerably more complicated, because then the nominal interest rate is exactly zero and the cash-in-advance constraint might not hold as an equality. In that case, the price level turns out to be not uniquely pinned down.

For $q > \beta - 1$ we have assumed that the cash-in-advance constraint is always binding (holds as an equality); in the resulting equilibrium that we computed the interest rate is positive, which is consistent with this fact, but you may wonder whether there are other equilibria in which this is not the case. It can be proved that there are no equilibria in which the cash-in-advance constraint is slack.[10].

We derived a unique possible price sequence using the conditions for a competitive equilibrium, but do we know that they are all satisfied? The answer is yes. You can check (and will do it in the problem set) that, once the Friedman distortion (2.11), the cash-in-advance constraint (2.12), the Fisher equation (2.10), and the transversality condition (2.9) hold, the allocation is optimal from the perspective of the households. The only equation that we have not checked is the transversality condition. Using the Fisher equation to substitute out the interest rate and (2.16) for M_t/P_{t-1}, this condition simplifies to

$$\liminf_{t\to\infty} \beta^t \frac{P_0}{1+R_0}\frac{M_t}{P_{t-1}} = \liminf_{t\to\infty} \beta^{t+1}\frac{P_0}{1+R_0} = 0.$$

If this were the end of the story, our course would be (almost) over. Unfortunately, there are several complications:

- First, our simple model features a very stable money demand. With constant inflation (and constant nominal interest rates), households desire to consume a constant stream of cash goods, so that real balances M_t/P_t remain constant and money and prices grow at the same rate. When the demand for cash goods is subject to shocks, setting M_t on a constant-growth path would *not* imply that P_t follows along, but rather P_t would be buffeted by

[9] In reality, you probably heard that several central banks have set slightly negative nominal interest rates on the excess reserves that banks hold with them. This is possible because storing currency has a cost.

[10] You need to work with the transversality condition to do so, and we will not do it here; you can take my word for it, or work out the algebra yourself...

the shocks to money demand. In practice, the demand for real money is unstable, so this strategy would deliver poor control of inflation, at least in the short run. This is the reason why central banks did not pay much attention to money supply between the mid 1980s until interest rates dropped to zero.[11]

- Even on a theoretical level, the assumption of no government bonds is more important than it seems at first. It is important for our proof that the transversality condition holds. Different choices of fiscal policy may clash with the chosen path for money and fail to deliver an equilibrium.

- Finally, the logarithmic case is very special. In particular, it turns out to be the case that other equilibria emerge for sure when $u(0)$ is finite. This is troublesome, because there are good reasons to believe that $u(0)$ is finite: if dollars did not exist, people could still use pounds, euros, yen, gold, cigarettes, shells or any other storable good to trade. If all else fails, they could barter; it would be very inefficient, but unlikely to deliver "infinitely negative" utility.

In conclusion, pure money supply rules are successful at pinning down inflation only in special circumstances. In the next chapters we will thus explore alternatives.

[11]The ECB is an exception, in that it officially it has always paid attention to the growth in monetary aggregates. Nonetheless, in practice even the ECB has used the interest rate as its primary instrument of monetary policy and the inflation rate and macroeconomic conditions as its primary gauges for the required policy adjustments.

Chapter 3

Interest Rate Targets in a Cash-in-Advance Economy

3.1 Introduction

In our last chapter, we studied the performance of money supply rules. We saw that they allow for a tight control of inflation only under very special circumstances. When money demand is unstable, money supply rules lead to unstable inflation. Moreover, for reasonable preference specifications, they cannot rule out paths where real money balances gradually disappear over time.

We will study here what happens if monetary policy is conducted by means of interest-rate rules. Rather than controlling the *quantity* of money supplied, the CB sets the *price* (the interest rate) and lets the quantity adjust based on demand.

Studying interest-rate rules is important because they have been the main instrument of monetary policy for essentially as long as we have had central banks.[1] This is particularly true of the period between the mid-1980s and 2008: the period after the inflation of the 1970s was conquered and until the Great Recession led interest rates to be at or close to the zero lower

[1] One reason the Federal Reserve System was created in the United States was to avoid the seasonal fluctuations in interest rates that were present before the United States had a central bank.

bound, which forced CBs to look at alternative instruments. While in the 1970s you could see many references to monetary aggregates in the central banks' policy statements, these all but disappeared since then.[2] If you read economic articles about the Bank of England these days, the main question is when it will raise interest rates.[3]

3.2 An Interest Rate Peg

We begin our analysis from the simplest of interest-rate rules: a fixed interest rate peg, where the CB sets the nominal interest rate at $R_t = \bar{R}$ in every period, and supplies any amount of money demand by the private sector.

Money supply has an effect on the government budget constraint:

$$B^S_{t-1} + M^S_{t-1} - T_t = \frac{B^S_t}{1+\bar{R}} + M^S_t. \tag{3.1}$$

We assume that, upon impact, any movements in the supply of money M^S_t are matched by corresponding movements in the supply of bonds, so as to maintain equality. This is the relevant case in practice: CBs increase or decrease the money supply by engaging in open-market operations, where they trade money for bonds.

Notice that a shift from bonds to money has some public-finance implications for the government, because debt pays interest and money does not. Sooner or later, these shifts will interact

[2]The Bundesbank and the ECB have been an exception in this, maintaining a bit more emphasis on money supply, but even in their case interest rates have been much more prominent in their announcements.

[3]In practice, the world is slightly more complicated. Before 2008, even central banks that relied purely on interest rates in setting their monetary policy stance typically used control over money quantities to manage interest rates on a day to day basis, which blurs the distinction between the two rules. After the policy of quantitative easing, the large excess reserves held by commercial banks make it impossible for CBs to control quantities of money used for transactions (rather than purely held as reserves), and policy will consequently fit better our description of a pure interest rate rule, where the CB stands ready to trade money and bonds (in this case, excess reserves) at a fixed price (the interest rate). We will analyze the balance sheet of a CB in more detail later in the course. For the purpose of this lecture, assuming that the CB fixes R_t and lets M^S_t be whatever households demand will be fine.

with taxes and thus fiscal policy. This interaction is extremely important and we will go in great detail about it in future lectures, but we will leave it aside for now.

To compute the set of equilibria, we use once again the three key relations that we derived previously: the Friedman distortion, the Fisher equation, and the cash-in-advance constraint.

Start from the Friedman distortion:

$$u'(c_{1t}) = 1 + \bar{R}, \quad t \geq 0 \tag{3.2}$$

Since u is assumed to be strictly concave, there will be a unique level of consumption of the cash good c_{1t} that satisfies this equation. If the demand for cash goods is constant (as it is for us), consumption of cash goods will be constant. Otherwise, consumption will fluctuate with demand, but the marginal cost of consuming cash goods (which is the marginal cost of consuming credit goods plus the opportunity cost of holding money) will remain fixed; this seems an attractive property of the rule.

Consider next the Fisher equation:

$$\frac{P_{t+1}}{P_t} = \beta(1 + \bar{R}), \quad t \geq 0 \tag{3.3}$$

This equation states that inflation is constant too! As an example, if the goal is to achieve a constant price level, all that is needed is to set $\bar{R} = 1/\beta - 1$.

Finally, the cash-in-advance constraint tells us $M_t = P_t c_{1t}$. Suppose that we choose $\bar{R} = 1/\beta - 1$, so that inflation is zero and prices are constant. Then, with constant prices and constant consumption of cash goods, even money is constant. This seems a great success: we attained control over the composition of consumption, inflation, and money too, apparently with none of the complications that arise under money supply rules. We also have the added bonus that unstable money demand no longer threatens price stability: if $u()$ is not a constant function, but rather it bounces around over time, this effect is absorbed by the money supply rather than spilling over to interest rates and prices.

Is this success? Unfortunately, there is one very important element of a competitive equilibrium that is not pinned down: the initial price level, P_0. Once I know P_0, equation (3.3) can be solved for the entire future sequence of prices, but nothing tells us what P_0 should be. The initial cash-in-advance constraint tells us that $M_0 = P_0 c_{1\,0}$, and we know from (3.2) what $c_{1\,0}$ is, but this is not good enough, because money supply is infinitely elastic and so M_0 can be anything: if households expect a high price P_0, they will demand high M_0, and vice versa if they expect a low P_0, they will demand a low M_0, and the CB will accommodate their demand.

In our simple model, indeterminacy of P_0 is not a big problem. The households do not care about P_0, since the consumption of cash goods is pinned down independently of it, and so is the consumption of credit goods (since $c_{2t} = y - c_{1t}$). But, in the real world, there are many reasons why changes in P_0 would have effects. If there are nominal frictions (sticky prices), changes in P_0 would have real output consequences. If some households are debtors and other are creditors, a higher P_0 would be good news for the debtors and the creditors.

You may still take comfort from thinking that this is purely a time-0 problem, so maybe it causes some instability at the beginning, but then in the long run all is well. Unfortunately, this is again an artefact of our simplifying assumptions: specifically, it comes about because we assumed no uncertainty. We will not study an economy with uncertainty, but, to give you a flavor of what would happen there, equation (3.3) would change to

$$E_t \frac{P_t}{P_{t+1}} = \frac{1}{\beta(1+\bar{R})}, \quad t \geq 0.$$

The Fisher equation would pin down *expected* (inverse) inflation, but not whether inflation is always 0 (say) or -5% some of the time and +5% the rest of the time. In this sense, with uncertainty, every period is a bit like period zero: there is some freedom in restarting the price level, based on the whims of what people expect at that point.[4]

A pure interest-rate peg is frowned upon in modern central banking, because it is widely perceived as leading to price instability through the mechanism described above. Nonetheless, in other periods of time, interest-rate rules that were very unresponsive to economic conditions have been adopted. As an example, the U.S. Federal Reserve pursued a policy of interest-rate stabilization through World War II and until the beginning of the 1950s.

[4]In economics, this is known as a "sunspot" equilibrium: the price level may depend on the realization of some random variable that has nothing to do with the economic fundamentals and is used purely as a coordination device for expectations. This is called a sunspot equilibrium because the emergence of sunspots is used as an example of such a variable.

To conclude this section, notice that the description above does not include any discussion of the transversality condition. Do we know that the transversality condition always holds? To answer this question, we would need to discuss how taxes change over time, since they are important in determining the sum of money and bonds $M_t^S + B_t^S/(1+\bar{R})$, as you can see from equation (3.1); this sum is in turn what matters for the transversality condition. We will return to this question in future lectures; for now, we just assume that the transversality condition holds, so that no initial price level P_0 can be ruled out from imposing it.

3.3 Taylor rules

If modern central banks use interest-rate rules, but they shun pure interest-rate pegs, what do they do? Rather than setting the interest rate once and for all, they let the rate respond to economic conditions. In practice, if you try to fit a rule that describes how CBs set their interest rates, you will do well with a rule that depends on the following three elements:

- Past interest rates. For a variety of reasons, CBs do not like too jerky movements in interest rates, but rather they raise and lower rates smoothly over prolonged cycles.

- Inflation. CBs respond to higher inflation by raising interest rates. It is often said that it is desirable to raise rates more than one for one with inflation (the so-called "Taylor principle.") We will see shortly a reason for this.

- Output. CBs respond to stronger output by raising rates.

Purists (e.g., Woodford (1998)) call a rule that sets interest rates as a function of economic conditions a "Wicksellian" rule, since Wicksell studied these rules in the late 19th-early 20th century (Wicksell (1898, 1907)). They would then reserve the name "Taylor" rules to specific numerical rules that Taylor analyzed, such as that of Taylor (1993):[5]

$$R_t = 1.5\pi_{t-1} + 0.5y_{t-1} + 0.015,$$

[5] Taylor's complete specification is slightly more complicated, because it deals with averaging several quarters of data.

where y_{t-1} is the deviation of output from its trend.

Most practitioners and many academics will use the term "Taylor rules" for generic interest-rate rules that depend on economic conditions, and we will follow this terminology (while retaining deep respect for Wicksell's pioneering work).

We will focus on rules where interest rates depend on inflation only. This is for several reasons:

- Inflation is by far the element that is stressed the most in the literature, and is the main way to achieve local determinacy/indeterminacy (we will define this below);
- Responding to output in our model is not interesting because we would need a model of short-run frictions to have interesting output movements;
- The dependence on past interest rates complicates the algebra but does not affect determinacy results (once the implied long-run response is computed).

Specifically, we will consider rules of the following type:

$$R_{t+1} = \frac{\bar{\pi}^{1-\alpha}}{\beta} \left(\frac{P_t}{P_{t-1}} \right)^{\alpha} - 1, \tag{3.4}$$

where $\bar{\pi}$ is some target inflation rate and α is a parameter that is typically assumed to be greater than 0. At first blush, this rule may seem somewhat complicated. However, our model yields nicer expressions after taking logarithms, and this rule is written in a way that it too will look nice after we take logarithms. With $\alpha > 0$, this rule states that the nominal interest rate is an increasing function of the inflation last period (the most recent inflation observation available to the central bank). Define (one plus) inflation as $\pi_t := P_t/P_{t-1}$.

To study the implications when the CB sets the nominal interest rate according to (3.4), we combine this equation with the Fisher equation

$$\pi_{t+1} = \beta(1 + R_{t+1}) \quad t \geq 0. \tag{3.5}$$

We obtain

$$\pi_{t+1} = \bar{\pi}^{1-\alpha} (\pi_t)^{\alpha} \quad t \geq 0, \tag{3.6}$$

which is a difference equation in inflation. Unless the transversality condition intervenes (which we will consider in future lectures), we have no boundary conditions: any sequence of inflation rates satisfying (3.6) will be part of an equilibrium, starting from an arbitrary choice of P_0 (and thus π_0). To check this, start from a sequence that satisfies (3.6). From this sequence, equation (3.4) tells us what the nominal interest rate will be (except we need an initial condition for R_0, which we take as exogenous). We can then use the Friedman distortion (2.11) to compute the consumption of cash goods, and finally the cash-in-advance constraint tells us what money has to be (assuming that it binds, i.e., that $R_t > 0$): $M_t = P_t C_{1t}$.

To study all possible inflation sequences that satisfy (3.6), we first check whether it admits a steady state. Replacing $\pi_{t+1} = \pi_t = \pi^{SS}$ we obtain

$$\pi^{SS} = \bar{\pi}^{1-\alpha} \left(\pi^{SS}\right)^{\alpha} \Longrightarrow \quad \pi^{SS} = \bar{\pi} :$$

there is a unique steady state, with inflation equal to the target (which is why I called $\bar{\pi}$ "target" in the first place).

Next, we study other equilibria. Outside of a steady state, it is much easier to work with the log of the inflation rate, so we take the log of equation (3.6), which yields

$$(\log \pi_{t+1} - \log \bar{\pi}) = \alpha(\log \pi_t - \log \bar{\pi}). \tag{3.7}$$

If $|\alpha| < 1$, there are many paths of inflation that satisfy (3.7), and they all converge to the steady state. We can use each of these paths to construct a competitive equilibrium, using the Friedman distortion and the cash-in-advance as above. This situation is called *local indeterminacy* (around the steady state): given any neighborhood of the steady state, there are infinite paths that start in the neighborhood and remain there for ever. While in a deterministic economy we can at least conclude that inflation would be at the steady state in the limit, this is no longer true once uncertainty is taken into account: there would be many sunspot equilibria where inflation is constantly buffeted around by the whims of expectations.

If $|\alpha| > 1$, we are in the opposite situation, *local determinacy.* $\alpha > 1$ corresponds to the case in which the CB raises interest rates more than one for one with inflation.[6] In this case, given

[6] More precisely, it raises $\log(1 + R_{t+1})$ more than one for one in response to $\log \pi_t$. However, for small inflation levels, the two statements are equivalent, since a linearization around $\pi_t \approx 1$ and $\bar{\pi} \approx 1$ would yield

any neighborhood of the steady state, there is a unique path that starts within the neighborhood and remains there: the steady state itself. By equation (3.7), any path that does not start at steady state would necessarily diverge (in logs) to either $-\infty$ or $+\infty$. One can also prove that, even with uncertainty, there is a unique path that is stable; all other paths would diverge, at least with positive probability.

Most practitioners are happy stopping at local determinacy, and assuming that a Taylor rule that yields a locally determinate equilibrium (in our case a steady state) pins down inflation uniquely. They are convinced that equilibria where inflation just "blows up" are implausible. However, in our readings we saw examples of hyperinflations, so we will not be happy ruling them out simply as implausible.

If we study these explosive paths in more detail, it turns out that their validity depends on whether we start above or below steady state.

If $\pi_0 > \bar{\pi}$, we obtain a sequence of inflation rates that is exploding to $+\infty$; we can then use the Friedman distortion and the cash-in-advance constraint to find the associated allocation, and we obtain a valid equilibrium path (as usual, assuming that the transversality condition holds).

If $\pi_0 < \bar{\pi}$, we can proceed as in the case of $\pi_0 > \bar{\pi}$, but we hit a snag. The difference equation (3.7) would imply that $\log \pi_t \to -\infty$, i.e., $\pi_t \to 0$. However, if π_t gets sufficiently close to zero, equation (3.4) would imply a negative nominal interest rate! If we accept as possible for the central bank to commit to deliver negative nominal interest rates, then this rule would exclude paths where $\pi_t \to 0$ (remember that this means $P_t/P_{t-1} \to 0$, i.e., inflation as conventionally defined converges to -100%).

There are some theoretical papers that argue that this commitment is possible in principle (including one of my own, Bassetto (2004)); whether this is theoretically possible or not, in practice a central bank would never do it, because it is equivalent to offer free money to people in that state of nature, which would destroy the value of money. Alternatively (and more plausibly), we can incorporate the zero-lower bound by changing our Taylor rule. As an example, we could

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$R_{t+1} \approx (1-\alpha)(\bar{\pi}-1) + \alpha(\pi_t - 1)$.

change it to

$$R_{t+1} = \max\{0, \frac{\bar{\pi}^{1-\alpha}}{\beta} (\pi_t)^{\alpha} - 1\}. \tag{3.8}$$

In a very influential paper, Benhabib et al. (2001) show that any fix (not just the one above) leads to the emergence of a second steady state, which is locally indeterminate. The zero lower bound is thus a further hindrance to achieving control of inflation by using simply interest-rate rules.

3.4 Taylor Rules: Targeting Rules or Reaction Functions?

In equation (3.4), the nominal interest rate depends on past inflation. In the economics literature, there are many papers that write rules in which the nominal interest rate depends on *current* inflation, or even on future inflation expectations. Consider the following rule:

$$R_{t+1} = \frac{\bar{\pi}^{1-\alpha}}{\beta} \pi_{t+1}^{\alpha} - 1. \tag{3.9}$$

Substituting the rule into the Fisher equation (3.5) (and neglecting the problem of the zero lower bound), we obtain

$$\pi_{t+1} = \bar{\pi}^{1-\alpha} (\pi_{t+1})^{\alpha} \Longrightarrow \pi_{t+1} = \bar{\pi} :$$

inflation is uniquely pinned down, with the notable exception of P_0.

Is this a successful way of achieving price stability? To understand the answer to this question, it is useful to introduce a distinction due to Svensson and Woodford (2005). They define a *targeting rule* as "describ[ing] conditions that the forecast paths must satisfy in order to minimize a particular loss function" and a *reaction function* as "specify[ing] the central bank's instrument as a function of predetermined endogenous or exogenous variables *observable to the central bank at the time that it sets the instrument*" (emphasis added).

A targeting rule describes a relationship among endogenous variables that is satisfied in equilibria that have potentially desirable properties. A CB may find such a rule desirable for communications: just as a CB may announce its target inflation, it may announce a targeting rule to describe how inflation and interest rates should ideally be connected. This may seem useless in our simple economy, where the CB might as well announce directly what is the desirable level of inflation. But, in a complicated world with many shocks, the CB will not want inflation to be constant, and more sophisticated representations such as (3.9) may be needed to characterize the CB *wishes*.

A targeting rule is *not* a description of how the central bank operates to get to its objective, because the CB does not know π_{t+1} when it is setting R_{t+1}. To know how the CB goes about achieving its objective, we need to know about its reaction function.

So, while equations as (3.9) may be satisfied by a unique equilibrium, they do not represent a description of the way the CB is successful in attaining it.

3.5 Conclusion

Just as in the case of money-supply rules, interest-rate rules by themselves are not successful at ensuring a unique equilibrium with low and stable inflation. In the next chapters, we will bring fiscal policy to the forefront of our analysis, and we will see how fiscal policy can help achieving what "pure" monetary policy could not.[7]

[7] Defining what is *monetary* and what is *fiscal* policy in practice is extremely difficult, because the two are intimately linked through the government budget constraint.

Chapter 4

Bringing Fiscal Policy into Monetary Policy

4.1 Introduction

In the empirical papers that we discussed in class, fiscal policy played a prominent role. In contrast, taxes were so far an afterthought in our theoretical analysis: they were present, but they did not seem to do very much. We hinted that they played an important role through the transversality condition, but we also did not pay that much attention to this condition either, mostly assuming that it was satisfied.

In the next couple of chapters, the connection between monetary and fiscal policy will finally take center stage, and the government budget constraint that links the two will get its due prominence.

Fiscal policy will help in our quest to find conditions under which an equilibrium is unique: after failing to achieve this with either monetary policy rules or interest rate rules by themselves, we will see that adding fiscal policy to the mix can help. However, this help will come at a cost: if pinning down the price level can only be done with help from fiscal policy, then a truly independent central bank never exists.

4.2 The Household Present-Value Budget Constraint (PVBC)

So far, we have written the household and government budget constraints in their flow form. In the case of the households, this was given by the sequence expressed in (2.2) with the initial time-0 version[1]

$$W_{-1} = T_0 + M_0 + \frac{B_0}{1+R_0}. \tag{4.1}$$

In this chapter, it will often be convenient to combine this sequence of budget constraints in a single one, the "present-value budget constraint" (PVBC). To derive it, combine first the period-0 budget constraint with the period-1 budget constraint, by substituting out B_0. This yields

$$W_{-1} = T_0 + \frac{1}{1+R_0}\left[T_1 - P_0(y - c_{1\,0} - c_{2\,0}) + R_0 M_0 + M_1 + \frac{B_1}{1+R_1}\right].$$

Proceed again substituting for B_1, B_2, up to B_J:

$$\begin{aligned} W_{-1} = T_0 + \sum_{s=0}^{J}\left[\prod_{v=0}^{s}(1+R_v)^{-1}\left(T_{s+1} - P_s(y - c_{1s} - c_{2s}) + R_s M_s\right)\right] \\ + \prod_{v=0}^{J}(1+R_v)^{-1}\left(\frac{B_{J+1}}{1+R_{J+1}} + M_{J+1}\right). \end{aligned} \tag{4.2}$$

Next, we know that the household transversality condition (2.9) must hold. The transversality condition is, strictly speaking, part of the household first-order conditions, not of its budget constraint. Nonetheless, it represents what is binding of the no-Ponzi condition.[2] The optimal plan of a household will be the same whether we impose the no-Ponzi condition or the transversality condition.

While the transversality condition requires the infimum limit to be zero, we will deal with fiscal policy rules that are sufficiently well behaved that we can replace the infimum limit with a regular limit in equation (2.9).[3] Taking limits as $J \to \infty$ in (4.2) and using (2.9) evaluated as

[1] Remember that what is special about time 0 is that, since we do not model period -1, we do not need to distinguish between B_{-1}, M_{-1}, or $P_{-1}(y - c_{1\,-1} - c_{2\,-1})$; we lump all these together in some initial nominal wealth W_{-1}, which, by assumption, is the same for all households.

[2] More precisely, the no-Ponzi condition is what forces households to set a plan such that the left-hand side of (2.9) is nonnegative. Optimality will then dictate that it is exactly equal to zero.

[3] Intuitively, this rules out fiscal policies where taxes follow explosive cycles.

a regular limit, we obtain the household PVBC:

$$W_{-1}+\sum_{s=0}^{\infty}\left[\prod_{v=0}^{s}(1+R_v)^{-1}P_s y\right]=T_0+\sum_{s=0}^{\infty}\left[\prod_{v=0}^{s}(1+R_v)^{-1}\left(T_{s+1}+P_s(c_{1s}+c_{2s})+R_s M_s\right)\right]. \quad (4.3)$$

The household PVBC replaces a sequence of budget constraints with a single one, evaluated at time 0. The left-hand side represents all sources of income for a household: its initial (nominal) wealth, W_{-1}, plus the present value of selling its endowment from period 0 all the way to infinity. The right-hand side represents a household's use of its resources. A household spends its income on paying taxes, buying consumption of cash and credit goods, and on holding money. Holding money is costly because of its opportunity cost, which is equal to the nominal interest rate that a household could have earned investing in bonds instead. Another useful way to understand this cost is to rewrite equation (4.3) using the cash-in-advance constraint. From the cash-in-advance constraint, we know that either $R_s = 0$ or $M_s = P_s c_{1s}$. With this, we can rewrite (4.3) as

$$W_{-1}+\sum_{s=0}^{\infty}\left[\prod_{v=0}^{s}(1+R_v)^{-1}P_s y\right]=T_0+P_0 c_{1\,0}+\sum_{s=0}^{\infty}\left[\prod_{v=0}^{s}(1+R_v)^{-1}\left(T_{s+1}+P_{s+1}c_{1\,s+1}+P_s c_{2s}\right)\right]. \quad (4.4)$$

This equation shows two things:

- Because of the opportunity cost of money, cash goods are effectively more expensive than credit goods, as long as $R_s > 0$, since they are discounted by one fewer interest factor. This is precisely the source of the Friedman distortion.

- The reason taxes have a different timing from consumption in equation (4.3) is that they have to be paid at the beginning of the period, whereas consumption payments are settled at the beginning of the *subsequent* period. For this reason, taxes act like a cash good: while purchased later, cash goods must be paid with money acquired at the beginning of the period, at the same time in which taxes are paid.

4.3 The Government PVBC

We can repeat the same steps that we did in the case of the households on the government budget constraint.

The generic period-t budget constraint for the government is equation (2.4). What is an asset for the households (money, bonds) is a liability for the government, and vice versa (taxes). In the government budget constraint, goods do not appear, since we assumed government spending away. The period-0 version is

$$W_{-1} = T_0 + M_0^S + \frac{B_0^S}{1+R_0}. \tag{4.5}$$

The households' initial nominal wealth, W_{-1}, is matched by an equal initial liability of the government. This has to be the case, since our economy is closed and there is no capital that can be saved from one period to the next.

Once again, we substitute the bond-supply B_0^S from equation (2.4) evaluated at $t = 1$ into (4.5) and obtain

$$W_{-1} = T_0 + \frac{1}{1+R_0}\left[T_1 + R_0 M_0^S + M_1^S + \frac{B_1^S}{1+R_1}\right];$$

we proceed recursively up to period $J+1$ and obtain

$$\begin{aligned} W_{-1} = T_0 + \sum_{s=0}^{J}\left[\prod_{v=0}^{s}(1+R_v)^{-1}\left(T_{s+1} + R_s M_s^S\right)\right] + \\ \prod_{v=0}^{J}(1+R_v)^{-1}\left(\frac{B_{J+1}^S}{1+R_{J+1}} + M_{J+1}^S\right). \end{aligned} \tag{4.6}$$

In the case of the households, our next step was to take the limit as $J \to \infty$ and invoke the transversality condition. However, as we noted, the transversality condition was coming from the no-Ponzi condition that prevented households from going into unlimited debt. The government does not face a similar problem, for two reasons:

- We did not impose any upper limit on taxes. So, a government with very large debt could set very large taxes to pay for it. This is different from the households, that have a fixed income in each period.

- Government debt is just a promise to deliver money, and the government is free to print money as necessary (something that households of course cannot do).

The first reason is implausible in practice. We will assume that the government faces an upper bound in its ability to raise real taxes. In real life, raising revenues comes with distortions:

there are administration and enforcement costs, not everybody is the same and so not everybody would be able to pay astronomical amounts of taxes, even if there were enough wealth in the aggregate to pay them, and discriminating across people by taxing the rich more would introduce costly distortions in the household behavior.

However, the second reason will remain valid, except in some specific circumstances. For example:

- We assumed nominal debt issued in the home currency. The transversality condition will apply when a country issues debt denominated in a foreign currency. This is very common in the case of developing economies that struggle with credibility and would be too tempted to use inflation as a cheap way to avoid raising taxes ex post. Similarly, the transversality condition holds for the Treasury departments of countries in the Eurozone, since they cannot print Euros at will.

- A transversality condition would apply if the central bank is committed to a money-supply rule. A money-supply rule restricts the ability to just print money to repay government debt. However, at least if the money supply is increasing over time, even with a money supply rule or foreign-denominated debt, government solvency would only require

$$\lim_{t\to\infty} B_t \prod_{s=0}^{t} (1+R_s)^{-1} \leq 0 :$$

  the condition would only apply to government bonds, not to money itself, since there is no commitment by the government to ever offer anything in exchange for money.[4]

4.4 The Fiscal Theory of the Price Level

Even if the government does not have a transversality condition, it would appear that we have another way of attaining the same result. Specifically, start from the household transversality

[4] We will discuss the case of increasing money supply more in future lectures. I will call it then the case of "fiat" money: money has value in this case just because of its transaction role, but by a promise to repurchase it for gold or some other real asset.

condition (2.9), and impose market clearing (one of the conditions for competitive equilibrium), which implies $M_t = M_t^S$ and $B_t = B_t^S$. We obtain

$$\liminf_{t\to\infty}\left(\frac{B_t^S}{1+R_t} + M_t^S\right)\prod_{s=0}^{t-1}(1+R_s)^{-1} = 0, \tag{4.7}$$

which is precisely the condition that we need to obtain a PVBC from the limit of (4.6):

$$W_{-1} = T_0 + \sum_{s=0}^{\infty}\left[\prod_{v=0}^{s}(1+R_v)^{-1}\left(T_{s+1} + R_s M_s^S\right)\right]. \tag{4.8}$$

When this condition holds, it has an interpretation similar to that of the households. The initial nominal liabilities of the government, W_{-1}, must be equal to the present value of current and future tax revenues, plus *seigniorage* (or the present value of the "inflation tax"), the gain that the government obtains when households hold money rather than bonds, thereby offering to the government the benefit of an interest-free loan.[5] Seigniorage can be rearranged in an alternative expression, that is equivalent but may useful to gain a different perspective, and thus more economic intuition:

$$\begin{aligned}
&\sum_{s=0}^{\infty}\left[\prod_{v=0}^{s}(1+R_v)^{-1}R_s M_s^S\right] = \\
&\sum_{s=0}^{\infty}\left[\prod_{v=0}^{s}(1+R_v)^{-1}\left((1+R_s)M_s^S - M_s^S\right)\right] = \\
&\sum_{s=0}^{\infty}\left[\prod_{v=0}^{s}(1+R_v)^{-1}(M_{s+1}^S - M_s^S)\right] + M_0^S.
\end{aligned}$$

In this form, we see that seigniorage represents the present value of the money that the government prints.

You may wonder whether it matters that we did not impose the transversality condition on the government directly, but relied on the households' condition and market clearing. The answer turns out to be yes: it is very important. Market clearing only has to be satisfied at *equilibrium* prices and interest rates. It is *not* a constraint that the government has to satisfy for arbitrary

[5] Note that seigniorage is strictly positive as long as the nominal interest rate is. In the past lectures, we studied equilibria in which nominal interest rates are strictly positive and the price level is constant. So, there is an inflation tax even when inflation is zero, as long as the opportunity cost of holding money is positive!

price and interest rate sequences. In other words, if (4.8) is violated, it may be that *prices* will have to adjust to ensure that the demand of money and bonds is equal to their supply, rather than the government's having to adjust its *taxes*.

Whether prices or taxes will ensure that (4.8) is satisfied will depend on the government choice of monetary/fiscal policy regime. The situation in which it is prices that adjust is called the *fiscal theory of the price level* (FTPL).

To see the power of the FTPL in action, we reconsider the case in which monetary policy sets a fixed interest rate $\bar{R}$, independently of what happened in the past. We saw last time that, neglecting the transversality condition, this meant that any initial price level P_0 would be an equilibrium, and that sunspot equilibria would emerge under uncertainty. Suppose now that fiscal policy is set as follows: T_0 is exogenously given, and

$$T_t = \bar{T} P_{t-1}, \tag{4.9}$$

for some constant level $\bar{T}$. Is it still true that any initial price level can be an equilibrium? To find out, we need to check whether (4.8) holds.[6] Substituting the monetary and fiscal policy rule into equation (4.8), we obtain

$$W_{-1} = T_0 + \sum_{s=0}^{\infty} \left[(1+\bar{R})^{-(s+1)} \left(P_s \bar{T} + \bar{R} M_s \right) \right]. \tag{4.10}$$

Assume that $\bar{R} > 0$. Then, in a competitive equilibrium, the cash-in-advance and Friedman-distortion condition will hold, and so

$$M_s = P_s c_{1s} = P_s \bar{c}, \text{ where } \bar{c} := (u')^{-1}(1+\bar{R}).$$

Substituting this into (4.10), equilibrium would require

$$W_{-1} = T_0 + \sum_{s=0}^{\infty} \left[(1+\bar{R})^{-(s+1)} P_s \left(\bar{T} + \bar{R}\bar{c} \right) \right]. \tag{4.11}$$

[6] Equation (4.8) comes from the government budget constraint, which determines how many bonds the government supplies, from market clearing, which will hold in an equilibrium, and the transversality condition. So, in a candidate equilibrium, whether (4.8) holds or not hinges on whether the transversality condition holds or not. By working with the government budget constraint to obtain $B_t^S/(1+R_t) + M_t^S$, you could verify directly the conditions under which the transversality condition holds, as is done in section 4.5 in proving that the fiscal rule (4.9) is active.

Next, the Fisher equation says that, in equilibrium,

$$P_s = \beta(1+\bar{R})P_{s-1}, \quad s \geq 1 \Longrightarrow P_s = \beta^s(1+\bar{R})^s P_0, \quad s \geq 0;$$

we use this to substitute all future prices in (4.11) and find that whether the transversality condition will hold at a candidate equilibrium depends on whether the following is true

$$W_{-1} = T_0 + \sum_{s=0}^{\infty} \left[\beta^s(1+\bar{R})^{-1} P_0 \left(\bar{T} + \bar{R}\bar{c}\right)\right] \tag{4.12}$$

In equation (4.12):

- W_{-1} is given as an exogenous initial condition;
- T_0 is also exogenously given;
- likewise, $\bar{T}$ and $\bar{R}$ are exogenously given, and $\bar{c}$ is uniquely determined by $\bar{R}$.

It follows that there will be at most one price level P_0 for which (4.12) will hold. This price level can be solved as follows:

$$P_0 = \frac{(1+\bar{R})(W_{-1} - T_0)(1-\beta)}{\bar{T} + \bar{R}\bar{c}}. \tag{4.13}$$

The solution is valid provided that $P_0 > 0$, which requires $\text{sign}(W_{-1} - T_0) = \text{sign}(\bar{T} + \bar{R}\bar{c})$ (and, of course, $\bar{T} + \bar{R}\bar{c} \neq 0$). The economic intuition behind the fiscal theory of the price level is the following. At the beginning of period 0, the government starts with an initial amount of nominal liabilities, net of taxes, equal to $W_{-1} - T_0$. Fiscal policy is chosen so that the present value of taxes is a fixed *real* amount.[7] Similarly, the interest-rate policy turns seigniorage in each period into a fixed real amount ($\bar{R}\bar{c}$). Then, the price level must be equal to the ratio of the nominal liabilities of the government to the present value of real resources (taxes and seigniorage) that are committed to the repayment of those liabilities. In this case, the price level is as much the

[7] Due to the cash-in-advance timing, when converting taxes from nominal to real amounts, it is more convenient to think of T_t as a $t-1$ variable, which is then deflated by P_{t-1}. This is convenient for economic intuition, for otherwise real taxes would not be exactly fixed, but the results of course are independent of this definition of what "real" taxes are. While T_t/P_t is not independent of prices, nonetheless the FTPL holds here, because a unique price level P_0 would ensure that the present value of government revenues is equal to its liabilities.

inverse of the value of *bonds* as it is the inverse of the value of *money*. This is not surprising, because the interest rate peg includes a commitment to deliver money in exchange for maturing bonds at par, so that we know that money and maturing bonds must have the same value!

In our previous chapter, P_0 was not pinned down because there were equilibria with high prices and a high money supply and equilibria with low prices and a low money supply. This is no longer the case, because the monetary/fiscal policy combination implies that different price levels will be associated with different levels of real resources committed to repayment of government debt. Consider the following thought experiment. Suppose P_0 starts above the equilibrium level (4.13) and consider the usual case in which the government starts off as a net debtor, i.e., $W_{-1} - T_0 > 0$. If the price level is above (4.13), it means that the present-value of resources available to the government to repay its debt is greater than the value of its debt. But, since we are in a closed economy, the flip side must be that the household wealth is smaller than the present-value of their consumption plan: at that price, households would be forced to cut back on their consumption, generating an excess supply of goods. This excess supply would tend to depress the price level, up until P_0 drops to the level given by (4.13).[8]

Just as the FTPL is successful at pinning down a unique equilibrium price level, it can be proved that it also eliminates all of the sunspot equilibria that arise in an economy with uncertainty. In fact the FTPL is the *only* complete and coherent theory that determines a unique price level.[9]

The FTPL comes with some unpleasant "side effects."

- Since fiscal policy is so important in delivering a unique equilibrium, there is not much scope for central bank independence. At the end of the day, it is Treasury that calls the

[8] Of course, while this thought experiment is useful to gain intuition, remember that our economy is *always* in equilibrium: we do not model the actual process by which prices adjust to achieve demand=supply.

[9] This is a bold statement, that other economists might dispute. There are other models that deliver uniqueness, but in each case I would argue that they are based on implausible assumptions. As an example, recall that we found a unique equilibrium under a money supply rule and logarithmic preferences for cash goods, in which case household would be willing to give up infinite resources to obtain cash as the nominal interest rate becomes arbitrarily large.

shots.

- Fiscal news, both about current and about future surpluses, have immediate consequences on prices. As equation (4.13) shows, any change in future expected taxes ($\bar{T}$) will have an immediate effect on the price level in period 0. Higher taxes will lower the equilibrium price level, because higher taxes imply a higher real value of debt repayments and thus require a lower price level to increase the real value of debt and restore equilibrium. To follow the analogy pushed by Sargent (1986) (and, more recently, by Cochrane (2005)), the price level acts as the (inverse of) the price of government debt, which is a claim to all future surpluses; in this sense, it is similar to the price of Microsoft stock, which is a claim to all future Microsoft profits. This analogy is somewhat disturbing, because the prices of company stocks are extremely volatile.[10] If we relied purely on the FTPL for price level determination, we would thus expect inflation to be extremely volatile. In a world of sticky prices, where inflation cannot be so volatile, we would then expect output to absorb the shocks, and thus the FTPL would imply a very unstable economy.

Fortunately, both of these side effects can be weakened somewhat. As an example, Del Negro and Sims (2015) study more complicated rules, where the usual monetary policy (Taylor rules) is used to attain local price determinacy, and fiscal "backing" plays a role only in ruling out extreme outcomes, such as the possibility that money may gradually lose all of its value. In these strategies, CBs still face a meaningful day-to-day problem; their independence may also be important on a day-to-day basis, to prevent political interference from interfering with the pursuit of an inflation target. The day-to-day management may ensure that the price level is not driven purely by expectations about future surpluses, and so may bring price stability. At the same time, even these independent CBs must face the reality that fiscal policy is essential for their long-run success, and that, ultimately, successful control of inflation cannot be attained unless monetary and fiscal policy properly cooperate.

[10] This volatility is in part explained by news about future company profits, but in large part it is left in the residual "risk premium."

4.5 Active and Passive Rules

Leeper (1991) proposed a useful classification of monetary and fiscal policy. Consider first the case of monetary policy. As we studied previously, Taylor rules that respond strongly to inflation turn the Fisher equation into a divergent difference equation, where only one equilibrium is stable (in our case, the steady state). These rules (that were associated with $\alpha > 1$ in our math) satisfy the Taylor principle of making the interest rate respond more than one for one to inflation. We call a monetary policy rule *active* if, when combined with the Fisher equation, it gives rise to an unstable difference equation; conversely, we call it *passive* if it gives rise to a stable difference equation, with a continuum of paths converging to steady state.

We wish to develop a similar language for fiscal policy rules. In the case of fiscal policy rules, the relevant difference equation will come from the government budget constraint. Define $H_t := \prod_{v=0}^{t-1}(1+R_v)^{-1}\frac{B_t^S}{1+R_t} + M_t^S$, with the convention for $t = 0$ that $\prod_{v=0}^{-1}(1+R_v)^{-1} := 1$ (the empty product is 1). In words, H_t is the value of nominal liabilities that the government sells in period t (period-t money supply, plus the bonds issued in period t and payable in period $t+1$), discounted to period 0. To obtain a difference equation for this variable, start from the government budget constraint (2.4) and rearrange it as follows:

$$\frac{B_t^S}{1+R_t} + M_t^S = \left(\frac{B_{t-1}^S}{1+R_{t-1}} + M_{t-1}^S\right)(1+R_{t-1}) - R_{t-1}M_{t-1}^S - T_t, \quad t > 0;$$

next, multiply both sides by $\prod_{v=0}^{t-1}(1+R_v)^{-1}$:

$$\prod_{v=0}^{t-1}(1+R_v)^{-1}\left(\frac{B_t^S}{1+R_t} + M_t^S\right) =$$
$$\prod_{v=0}^{t-2}(1+R_v)^{-1}\left(\frac{B_{t-1}^S}{1+R_{t-1}} + M_{t-1}^S\right) - \prod_{v=0}^{t-1}(1+R_v)^{-1}\left(R_{t-1}M_{t-1}^S + T_t\right), \quad t > 0;$$

finally, substitute the definition of H_t:

$$H_t = H_{t-1} - \prod_{v=0}^{t-1}(1+R_v)^{-1}\left(R_{t-1}M_{t-1}^S + T_t\right), \quad t > 0. \tag{4.14}$$

Period 0 is special, since (2.4) is replaced by (4.5). We then get

$$\frac{B_0^S}{1+R_0} + M_0^S = W_{-1} - T_0 \Longrightarrow H_0 = W_{-1} - T_0, \tag{4.15}$$

which acts as an initial condition for the difference equation. From the transversality condition (2.9), we know that a competitive equilibrium requires $\lim_{t\to\infty} H_t = 0$. In analogy with what we did for Taylor rules, we define *active* a fiscal policy rule that, combined with the difference equation (4.14) and the other competitive equilibrium conditions, yields a unique solution in which H_t converges to 0. We define the fiscal rule *passive* if instead it yields a continuum of solutions converging to 0.

This definition is best illustrated with a couple of examples.

In both examples, we assume that $\lim_{c\to 0} u(c) > -\infty$. We already discussed that this is plausible, because the absence of money, while undesirable, should not represent a complete collapse of the economy. Under this condition, it can be proven that

$$\lim_{R\to\infty} R(u')^{-1}(1+R) < \infty$$

and thus that we can find $\bar{S}$ such that $R(u')^{-1}(1+R) < \bar{S}$ for all values of R.

The economic content of the assumption above is to ensure that revenues from seigniorage (the inflation tax) remain finite. The case in which money is so valuable that households would be willing to devote potentially infinite resources to acquiring it is in practice irrelevant, so this assumption is plausible. With this assumption, we can prove that the following fiscal rule is passive: T_0 is given, and

$$T_t = \gamma(M_{t-1}^S + B_{t-1}^S), \quad t > 0, \tag{4.16}$$

with $\gamma \in (0,1)$.[11] According to this rule, Treasury sets taxes to cover a fixed fraction of the government liabilities.

Proof that the fiscal rule (4.16) is passive. Substitute (4.16) into the difference equation

[11] We could consider $\gamma \in (1,2)$, but again it would be empirically irrelevant: it would correspond to a government that every other period is in debt, repays more than 100% of it, and cuts taxes to go back in debt in the subsequent period.

(4.14):

$$
\begin{aligned}
H_t =& H_{t-1} - \prod_{v=0}^{t-1}(1+R_v)^{-1}\left(R_{t-1}M_{t-1}^S + \gamma(M_{t-1}^S + B_{t-1}^S)\right) = \\
& H_{t-1} - \gamma\prod_{v=0}^{t-2}(1+R_v)^{-1}\left(\frac{B_{t-1}^S}{1+R_{t-1}} + M_{t-1}^S\right) + \gamma\prod_{v=0}^{t-2}(1+R_v)^{-1}\frac{R_{t-1}M_{t-1}^S}{1+R_{t-1}} - \\
& \prod_{v=0}^{t-1}(1+R_v)^{-1}R_{t-1}M_{t-1}^S = (1-\gamma)\left(H_{t-1} - \prod_{v=0}^{t-1}(1+R_v)^{-1}R_{t-1}M_{t-1}^S\right), \quad t>0.
\end{aligned}
\tag{4.17}
$$

Next, use the Fisher equation to substitute the interest rate factor:

$$\frac{P_{t+1}}{P_t} = \beta(1+R_{t+1}) \quad t \geq 0,$$

$$H_t = (1-\gamma)\left(H_{t-1} - (1+R_0)^{-1}\beta^{t-1}\frac{R_{t-1}M_{t-1}^S P_0}{P_{t-1}}\right), \quad t>0.$$

Next, use the cash-in-advance constraint and the Friedman distortion: either $R_{t-1} = 0$ or $M_{t-1}^S/P_{t-1} = c_{t-1} = (u')^{-1}(1+R_{t-1})$. Use also the technical assumption that $R(u')^{-1}(1+R) < \bar{S}$; we get

$$|H_t| \leq (1-\gamma)|H_{t-1}| + (1-\gamma)\beta^{t-1}(1+R_0)^{-1}P_0\bar{S}, \quad t>0.$$

Iterating on this equation,

$$
\begin{aligned}
|H_t| \leq& (1-\gamma)^2|H_{t-2}| + (1-\gamma)\beta^{t-1}(1+R_0)^{-1}P_0\bar{S} + (1-\gamma)^2\beta^{t-2}(1+R_0)^{-1}P_0\bar{S} \leq \ldots \leq \\
& (1-\gamma)^t|H_0| + (1+R_0)^{-1}P_0\bar{S}\sum_{s=0}^{t-1}\beta^s(1-\gamma)^{t-s} = \\
& (1-\gamma)^t\left[|H_0| + \frac{(1+R_0)^{-1}P_0\bar{S}\left(1-\left(\frac{\beta}{1-\gamma}\right)^t\right)}{1-\frac{\beta}{1-\gamma}}\right] = \\
& (1-\gamma)^t\left[|H_0| + \frac{(1+R_0)^{-1}P_0\bar{S}}{1-\frac{\beta}{1-\gamma}}\right] - \frac{\beta^t(1+R_0)^{-1}P_0\bar{S}}{1-\frac{\beta}{1-\gamma}} \rightarrow_{t\to\infty} 0.
\end{aligned}
$$

QED.

When fiscal policy is set according to the passive rule (4.16), H_t converges to zero no matter what the initial price level is, which means that the transversality condition is met independently of the price level.

An example of an active fiscal policy rule is the one we used to illustrate the FTPL: T_0 is exogenously given, and $T_t = \bar{T}P_{t-1}$, as in (4.9).

Proof that the fiscal rule (4.9) is active. Substitute (4.9) into (4.14):

$$H_t = H_{t-1} - \prod_{v=0}^{t-1}(1+R_v)^{-1}\left(\bar{T}P_{t-1} + R_{t-1}M_{t-1}^S\right), \quad t > 0.$$

Use the Fisher equation:

$$H_t = H_{t-1} - (1+R_0)^{-1}\beta^{t-1}P_0\left(\bar{T} + \frac{R_{t-1}M_{t-1}^S}{P_{t-1}}\right), \quad t > 0.$$

Use the cash-in-advance constraint and the Friedman distortion:

$$H_t = H_{t-1} - (1+R_0)^{-1}\beta^{t-1}P_0\left[\bar{T} + R_{t-1}(u')^{-1}(1+R_{t-1})\right], \quad t > 0.$$

We thus get

$$\begin{aligned} &H_t = H_0 - (1+R_0)^{-1}P_0\sum_{s=0}^{t-1}\beta^s\left[\bar{T} + \beta R_s(u')^{-1}(1+R_s)\right] = \\ &W_{-1} - T_0 - (1+R_0)^{-1}P_0\sum_{s=0}^{t-1}\beta^s\left[\bar{T} + \beta R_s(u')^{-1}(1+R_s)\right] \rightarrow_{t\rightarrow\infty} \\ &W_{-1} - T_0 - (1+R_0)^{-1}P_0\sum_{s=0}^{\infty}\beta^s\left[\bar{T} + \beta R_s(u')^{-1}(1+R_s)\right]. \end{aligned} \tag{4.18}$$

From equation (4.18) we observe that $H_t \not\rightarrow_{t\rightarrow\infty} 0$ unless P_0 is set at exactly the right value, which is given by[12]

$$\frac{(W_{-1} - T_0)(1+R_0)}{\sum_{s=0}^{\infty}\beta^s\left[\bar{T} + \beta R_s(u')^{-1}(1+R_s)\right]}.$$

QED.

[12] The technical condition that $R(u')^{-1}(1+R) < \bar{S}$ ensures that the infinite sum in (4.18) is well defined. P_0 will be positive provided $W_{-1} - T_0$ and the infinite sum have the same sign. As an example, this includes the natural case in which the government has some debts left after initial taxes ($W_{-1} - T_0 > 0$) and future taxes are positive ($\bar{T} > 0$).

Having drawn a parallel between fiscal and monetary rules that may cause explosive paths, it is also important to notice a difference. As we observed in the previous lecture, explosive paths that are caused by active Taylor rules satisfy all of our requirements for competitive equilibria. They may look implausible, but they are not ruled out by our theory. In fact, the main reason they look implausible is because the Taylor rule itself looks implausible: faced with ever-increasing inflation, would the CB blindly keep raising rates according to the Taylor rule, or would it rather attempt some other strategy to bring inflation under control?

In contrast, explosive paths driven by the fact that H_t does not converge to 0 for a given fiscal policy rule and initial price level are *not* competitive equilibria: the transversality condition is an equilibrium condition.

We can thus classify the set of equilibria that will emerge from various combinations of fiscal and monetary policy rules as follows:

	Fiscal Policy	
	Passive	Active
Interest-rate rule: Passive	Indeterminacy (continuum of stable eq.)	Unique eq.
Interest-rate rule: Active	Local uniqueness (cont. of eq., only one stable)	Unique eq., explosive

4.6 Conclusion

In this chapter we have explored the role of fiscal policy in achieving a unique equilibrium. We have finally succeeded in finding conditions under which the equilibrium is indeed unique, and discovered that taxes are at the heart of this result. Fiscal policy rules that deliver a unique equilibrium do so because they link the real value of nominal debt to the present value of future government revenues (taxes and seigniorage).

As a last remark, in our analysis we implicitly assumed that taxes are entirely set by Treasury, and that the central bank does not have real assets of its own. When the CB has some real revenues or real assets of its own (e.g., gold reserves), its position may be stronger, because it need not rely entirely on Treasury to ensure that money has positive value. The implications of this observation have been studied by Obstfeld and Rogoff (1983).

Chapter 5

Unpleasant Monetarist Arithmetic

5.1 Introduction

In our previous chapter, we studied the fiscal theory of the price level, and we saw how it provides a coherent story that uniquely pins down the price level. The main punch line of the fiscal theory is that money and maturing government debt are the same thing (under the conditions that we discussed) and that we can thus think of the price level as the inverse of the price of (maturing) government debt. We saw that the fiscal theory has important implications for how surprise news about future surpluses would affect the price level immediately, and we also briefly discussed possible ways in which monetary-fiscal coordination may be used to weaken the direct link from deficit/surplus news to the price level, leaving some latitude for monetary policy to achieve inflation stabilization on a day-to-day basis.

In the fiscal theory of the price level, seigniorage revenues play a secondary role; when present, they are lumped together with general revenues. It is the interplay of nominal debt and real tax revenues that takes center stage. In this chapter, which follows Sargent and Wallace (1981), the emphasis is reversed: nominal debt will play a secondary role, and seigniorage will take center stage. Sargent and Wallace wrote their piece in a period when inflation was hovering around the double-digit mark in the United States and was well over that in other countries (including the UK); seigniorage was a much more important source of government revenues back then than it has been since. At the moment, their piece is more relevant in thinking about high inflation countries (e.g., Venezuela, most likely Greece if it were to abandon the Euro) than it is about the UK or the US. That said, even between the mid-1980s and 2008, seigniorage has not been a completely trivial source of revenues for the governments of advanced economies. For all the talk of money disappearing, even before the financial crisis of 2008, monetary base was a significant fraction of GDP (this week's problem set is meant to offer you an opportunity to assess this). Since 2008, central banks have greatly expanded their balance sheet (as we will discuss in future lectures) and have been raking correspondingly larger profits. If/when these profits disappear or turn into losses, the questions underlying unpleasant monetarist arithmetic may become relevant once again.

"Unpleasant monetarist arithmetic" follows the general theme that Sargent pushes in the other articles that we have read, in which he drives a sharp distinction between individual actions of monetary and fiscal policy and *changes in regime*. It is meant to illustrate that a single action of tightening monetary policy may be ineffective on its own: without a regime shift, that action will have to be more than undone in the future.

5.2 The setup

In this chapter, we will assume that preferences for the cash good are given by

$$u(c) = \log c.$$

As we previously discussed, this assumption is not terribly realistic. However, it has the advantage of simplifying our exposition, because it implies a unique equilibrium if monetary policy pursues a money-supply rule. Multiple equilibria are a side show for what we will study today, so it is convenient to take them out of the picture and focus on the main point of the day.

With the log assumption on preferences, the Friedman-distortion equation becomes

$$\frac{1}{c_{1t}} = 1 + R_t, \quad t \geq 0. \tag{5.1}$$

The description of the equilibrium conditions coming from private-sector optimization are completed, as usual, by the Fisher equation (2.10) and the cash-in-advance constraint (which we take to hold with equality):

$$M_t = P_t c_{1t}, \quad t \geq 0. \tag{5.2}$$

We substitute (2.10) and (5.2) into equation (5.1), for $t > 0$:

$$\frac{1}{c_{1t}} = \frac{P_t}{\beta P_{t-1}} \Longrightarrow \beta P_{t-1} = c_{1t} P_t = M_t, \quad t \geq 1$$

We thus obtain

$$P_t = M_{t+1}/\beta, \quad t \geq 0: \tag{5.3}$$

by controlling the money supply, the central bank also controls the price level (and inflation) in our economy. The log assumption is responsible for this result, and it also comes with the added bonus that the expression for P_t is particularly simple.

We next turn to the government present-value budget constraint, which, as we previously saw, must hold in equilibrium:

$$W_{-1} = T_0 + \sum_{s=0}^{\infty} \left[\left(T_{s+1} + R_s M_s^S \right) \prod_{v=0}^{s} (1 + R_v)^{-1} \right]. \tag{5.4}$$

We assume that fiscal policy is described by two parameters: an initial level of nominal taxes, T_0, and a long-run level of real taxes $\bar{T}$, measured by deflating taxes by the most recently available price level:

$$T_t = \bar{T} P_{t-1}. \tag{5.5}$$

With a money supply rule, we already discussed that the fiscal theory of the price level cannot hold: we already determined the price level from the money supply rule, so there is no separate need to determine it from equation (5.4). Nonetheless, not every combination of taxes and money supply will satisfy (5.4), which will create the potential for a showdown between Treasury (setting taxes) and the CB (setting the money supply). We explore this conflict further by substituting (2.10) and (5.5) into (5.4); this allows us to get rid of R_t for $t > 0$. We obtain:

$$W_{-1} = T_0 + M_0^S + (1 + R_0)^{-1} P_0 \bar{T} \sum_{s=0}^{\infty} \beta^s + (1 + R_0)^{-1} P_0 \sum_{s=0}^{\infty} \beta^s \left(\frac{M_{s+1}^S - M_s^S}{P_s} \right).$$

In this expression, we explicitly labeled money as money supply (even though, of course, in equilibrium $M_t = M_t^S$), to emphasize that it is a quantity under the control of the CB. Next, using the equilibrium value of the price level (5.3), we can also get rid of prices for $t > 0$:

$$W_{-1} = T_0 + M_0^S + \frac{(1+R_0)^{-1} P_0 \bar{T}}{1-\beta} + (1+R_0)^{-1} P_0 \sum_{s=0}^{\infty} \beta^s \left(\beta - \beta \frac{M_s^S}{M_{s+1}^S} \right)$$

Finally, using (5.1) and (5.2) for period 0, we also eliminate the two remaining endogenous variables, P_0 and R_0, to obtain

$$W_{-1} = T_0 + M_0^S + \frac{M_0^S \bar{T}}{1-\beta} + M_0^S \sum_{s=0}^{\infty} \beta^{s+1} \left(1 - \frac{M_s^S}{M_{s+1}^S} \right). \tag{5.6}$$

This expression shows that, in equilibrium, the initial nominal liabilities of the government must be matched either by taxes or by seigniorage, which, with our simplifying log assumption, takes the simple form of a discounted sum of terms related to future money growth.

5.3 The policy experiment

Suppose that we start the economy in a given equilibrium, which is described by a sequence of money supply $\{M_t^S\}_{t=0}^{\infty}$ and tax parameters T_0 and $\bar{T}$ so that (5.6) holds. This policy is characterized by an equilibrium that we can easily compute going backwards through our substitutions: we obtain a sequence of prices from (5.3), a sequence of interest rates (for $t > 0$) from (2.10), a sequence of consumption levels (for $t > 0$) from (5.1), an initial consumption level c_0 from (5.2), and an initial interest rate R_0 from (5.1). Finally, that (5.6) holds will imply (with the appropriate substitutions) that the government present-value budget constraint (5.4) also holds.

In the equilibrium described above, inflation is given by

$$\pi_{s+1} = \frac{P_{s+1}}{P_s} = \frac{M_{s+2}^S}{M_{s+1}^S}, \quad s \geq 0. \tag{5.7}$$

Suppose that the central bank is unhappy with the level of inflation in some period t, that is deemed excessive. At first glance, it is very straightforward to fix this: all the central bank has to do is to lower money growth between periods t and $t+1$, while leaving unchanged money growth in all other periods. From equation (5.7), this will have the desired effect on period-t inflation, while leaving inflation in all other periods unchanged.

This policy change requires some other adjustment to ensure that (5.6) continues to hold, since otherwise the left-hand side will now be bigger than the right-hand side. If the Treasury is willing to adjust T_0 and/or $\bar{T}$, then the central bank will indeed have successfully lowered inflation. Notice that this is as much a success of fiscal policy than it is of monetary policy: while lower inflation was brought about by lower money growth, lower money growth was only possible because taxes were raised to make up for the budget shortfall.

What if the Treasury is unwilling to cooperate, so that T_0 and $\bar{T}$ do not change? In this case, the government budget constraint, in its form given by (5.6), shows that cutting money growth in period t can only be attained if the central bank is willing to *increase* money growth in some other period. This in turn implies that, while inflation is going down in period t, this comes at the expense of more inflation in some other period. Moreover, as we verify next, the increase in inflation will be greater the farther into the future is the period in which the central bank undoes its tightening. To see this, define money growth: $q_s := M_s^S/M_{s-1}^S$. Equation (5.6) becomes then

$$W_{-1} = T_0 + M_0^S + \frac{M_0^S \bar{T}}{1-\beta} + M_0^S \sum_{s=0}^{\infty} \beta^{s+1}\left(1 - \frac{1}{q_{s+1}}\right).$$

Suppose that q_t shrinks by a factor $k < 1$, from q_t to kq_t. Suppose further that this is compensated by an adjustment at some future date $\hat{t}$, by some factor $\hat{k}$. Budget balance requires

$$\beta^t \left[\left(1 - \frac{1}{q_t}\right) - \left(1 - \frac{1}{kq_t}\right)\right] = -\beta^{\hat{t}} \left[\left(1 - \frac{1}{q_{\hat{t}}}\right) - \left(1 - \frac{1}{\hat{k}q_{\hat{t}}}\right)\right].$$

Simplifying, this yields

$$\frac{\beta^t}{q_t}\left[\frac{1}{k} - 1\right] = -\frac{\beta^{\hat{t}}}{q_{\hat{t}}}\left[\frac{1}{\hat{k}} - 1\right] \tag{5.8}$$

If $k < 1$, $\hat{k} > 1$: inflation goes up in period $\hat{t} - 1$. Furthermore, the bigger $\hat{t}$ is, the bigger $\hat{k}$ has to be, to undo the effect of discounting (seigniorage revenues far into the future are worth less, so this has to be compensated by a greater increase in the revenues).[1]

[1] Equation (5.8) implies also that there is a limit to the CB's ability to postpone the adjustment: $1 - 1/\hat{k}$ is bounded by 1. Eventually, even infinite inflation would not be sufficient to make up for the initial drop in seigniorage in period t.

5.4 Conclusion

The simple model that we discussed in this chapter shows that an adjustment to lower inflation necessarily requires a corresponding fiscal adjustment (an increase in taxes, or, in a richer model, a cut in government spending). If fiscal policy is unwilling to cooperate, a CB that pursues a policy of temporary price stabilization will find that its attempts are fruitless and even counter-productive in the long run. This situation is often mentioned as a reason why many inflation stabilization plans failed in Latin America: while they brought inflation temporarily under control, they did not address the fiscal shortfalls that were the original source of undesirably loose monetary policy. Once government debt started growing, central banks were eventually called to finance deficits once again, and inflation took off once more.

In the case of log preferences, there is a simple relationship between money growth and inflation. This relationship is in general more complicated, and current inflation may be determined not just by money growth one period ahead, but by the expectations about the entire future path of money growth. By playing with this case, Sargent and Wallace (1981) provide an example in which the initial money contraction at t does not deliver lower inflation even at t: rather, inflation may increase in *all* periods!!

The policy conclusion of this chapter is that, when advising the CB of a country with a precarious fiscal situation, it is pointless to recommend an aggressive inflation stance, unless you are confident that this will trigger a fiscal adjustment. As an example, in the event of a "Grexit" (an exit from the Eurozone by Greece) it would be pointless for the central bank of Greece to pursue a tight-money policy; this would simply delay the day of reckoning and make the subsequent inflation all the more painful.

Chapter 6

Inflation and Government Deficits

6.1 Introduction

Up to here, we have discussed the role of fiscal policy and fiscal deficits in driving monetary policy and inflation. In this chapter, we explore the relationship between inflation and deficits from the reverse perspective. We will show that inflation has a tendency to increase fiscal deficits, and we will discuss the circumstances in which this is particularly the case. We will also see that government deficits, as usually reported in the press (e.g., in the statistics pages of *the Economist*), do not appropriately represent the true measure of fiscal imbalance in a country, and we will consider some adjustments that allow us to better represent the extent by which a country is digging itself deeper into debt or is emerging from fiscal straits.

In class, we discussed various means by which deficit measures can be manipulated. I advertised generational accounting, an approach advocated by Auerbach et al. (1991, 1994) and Kotlikoff (1992) that undoes some of the manipulation that can occur under the standard definitions of deficits. Generational accounting aims at measuring how much each cohort contributes to the government, net of the transfers that it receives back from the government. I will not cover this discussion in detail in this book, since it is tangential to our main interest. I will instead assume that current government spending, taxes, and transfers represent a proper account of the stance of fiscal policy over time, and will study how these interact with nominal and real interest payments on national debt.

6.2 Measures of government deficit

We will define *deficit* as the government spending on purchases and transfers plus interest payments on debt, minus taxes. Conceptually this definition is straightforward, but I will be more precise when we look at data, because defining what is government spending and what taxes are is not necessarily straightforward.

In equations, I will define the deficit D_t in a period t as

$$D_t := G_t - T_t + \frac{R_{t-1}}{1+R_{t-1}} B^S_{t-1}, \tag{6.1}$$

where all variables have the same meaning as in previous chapters, except for the newly introduced government spending, G_t. As in previous chapters, whether government spending is or is not present will not have any effect for our main points. Nonetheless, a chapter on government deficits that does not include government spending would look rather odd.

Compared to previous periods, in equation (6.1), I did not include seigniorage. We will discuss seigniorage extensively in future lectures. Here, I will simply lump it together with other tax revenues in T_t.

To verify that the last term of equation (6.1) represents indeed interest payments on debt in period t, recall that the government issues in period $t-1$ promises to repay B^S_{t-1} dollars at the beginning of period t. By selling these claims, the government receives $B^S_{t-1}/(1+R_{t-1})$ from creditors in period $t-1$. Interest payments in period t are the difference between what the government repays (B^S_{t-1}) and what the government received.

For this chapter, the algebra will look clearer if we measure debt at the moment that it is issued, rather than when it is repaid. Accordingly, define

$$\hat{B}_t := \frac{B^S_t}{1+R_t} :$$

$\hat{B}_t$ is the value of debt issued by the government in period t, to be repaid (with interest) in period $t+1$.

With this definition, equation (6.1) becomes

$$D_t = G_t - T_t + R_{t-1}\hat{B}_{t-1}, \tag{6.2}$$

which looks more familiar.

Our first decomposition of deficit distinguishes between *primary deficit* and interest payments. The primary deficit is simply total deficit minus interest payments, i.e., $G_t - T_t$. Primary deficits are a measure of whether resources are flowing from the creditors to the government, or vice versa. A government that is running a primary deficit is taking fresh resources from creditors, by selling more debt than would be needed to roll over existing obligations. A government that is in primary balance is not requiring fresh resources, but it is rolling over its entire debt, including interest. A government in primary surplus is making some payments to creditors and not rolling over all of its debt (even though its payments may not be enough to cover all of the interest on the debt).

Ignoring money and seigniorage (by including $M_t^S - M_{t-1}^S$ in tax revenues), the government budget constraint is

$$\frac{B_t^S}{1+R_t} = B_{t-1}^S - T_t + G_t$$

or, in terms of $\hat{B}_t$,

$$\hat{B}_t = \hat{B}_{t-1}(1+R_{t-1}) - T_t + G_t. \tag{6.3}$$

Substituting (6.2) into (6.3) we obtain

$$\hat{B}_t - \hat{B}_{t-1} = D_t : \tag{6.4}$$

the government deficit measures the evolution of the *nominal* value of issued debt.

6.3 Adjusting deficits for inflation

While the definition of D_t makes sense in a strict accounting interpretation, from an economic perspective the evolution of nominal debt is not particularly interesting. This is because "dollars" (or "pounds") are a unit of account whose value changes over time. But creditors do not value "dollars" or "pounds" per se; they invest in government debt to save. What will be important to them is how much their investment allows them to increase future consumption: of cars, houses, cell phones, food, travel,... not pieces of paper. What is of interest to creditors is thus not the change in *nominal* debt, but the change in *real* debt: how many more cars, houses, etc. can they buy with the new stock of debt?

In equations, rather than caring about $\hat{B}_t - \hat{B}_{t-1}$, we are interested in $\hat{B}_t/P_t - \hat{B}_{t-1}/P_{t-1}$. We get

$$\frac{\hat{B}_t}{P_t} - \frac{\hat{B}_{t-1}}{P_{t-1}} = \frac{\hat{B}_t}{P_t} - \frac{\hat{B}_{t-1}}{P_t} + \frac{\hat{B}_{t-1}}{P_t} - \frac{\hat{B}_{t-1}}{P_{t-1}} = \frac{D_t}{P_t} - \frac{\hat{B}_{t-1}}{P_{t-1}}\left(1 - \frac{P_{t-1}}{P_t}\right), \tag{6.5}$$

where the last step of equation (6.5) uses (6.4). To compute evolution of real debt, we start from real deficit (nominal deficit divided by the price level), but then we adjust it by the product of real debt in the previous period, times an inflation factor that measures how inflation has eroded the real value of a dollar from period $t-1$ to t. To see this last term at work, it's useful to look at special cases. When inflation is zero ($P_t = P_{t-1}$), we get

$$\left(1 - \frac{P_{t-1}}{P_t}\right) = 0 :$$

no adjustment is required, the real value of a dollar has not changed, so interest payments on existing debt are entirely a real payment to the creditors. In this case, the real deficit is the correct measure of the change in the real value of debt. When inflation is positive ($P_t > P_{t-1}$),

$$\left(1 - \frac{P_{t-1}}{P_t}\right) > 0 :$$

in this case, part of the interest payments on debt does not represent a change in real debt, but is instead needed to make up for the loss in real value of the principal. Hence, the change in the real value of debt is less than the real deficit.[1] In the case of deflation, the reverse occurs: the change in real value of debt is greater than the real deficit, because deflation increases the real value of existing debt even without interest payments.

The inflation factor involves the inverse of inflation, which is somewhat cumbersome. A quick

[1]This assumes that initial debt is positive, i.e., that the government owes to the private sector. Otherwise, the correction would go in the other direction: the government's assets would be the ones losing value.

and handy approximation when $P_t/P_{t-1} \approx 1$ is

$$1 - P_{t-1}/P_t \approx P_t/P_{t-1} - 1 :$$

with this approximation, the adjustment is simply the product of real debt from the previous period times inflation.

The equations above seem to suggest that inflation reduces the real value of the debt and deflation increases it. This is true *if we hold the deficit fixed,* and will typically happen when inflation (or deflation) comes as a surprise.

When inflation is expected by the creditors, there is a good reason to assume that the deficit is *not* fixed. Looking at equation (6.2), for low to moderate levels of inflation, it is plausible to assume that there is no strong link between real spending or real taxes and inflation, i.e., that G_t and T_t are proportional to P_t.[2] However, there is in general a strong link between nominal interest rates and inflation, through the Fisher equation: when inflation is high, savers require higher nominal interest rates to compensate. To discuss this more in detail, we need to go beyond the government budget constraint, the only equation that we have used so far, and embed it into a complete economic model where the determination of interest rates is endogenous. For this, we rely on the simple economic model that we have used throughout this course. In that model, a specific version of the Fisher equation, equation (2.10), holds exactly period by period. Using this equation, interest payments in period t will thus be

$$R_{t-1}\hat{B}_{t-1} = \left(\frac{P_{t-1}}{\beta P_{t-2}} - 1\right)\hat{B}_{t-1}.$$

We substitute this expression into equation (6.2) and find that, before adjustment, the deficit will be

$$D_t = G_t - T_t + \left(\frac{P_{t-1}}{\beta P_{t-2}} - 1\right)\hat{B}_{t-1} : \tag{6.6}$$

high inflation will be associated with high deficits, because the government faces a hefty interest rate bill. This additional deficit is not really a burden on future taxpayers, because the larger

[2] At very high inflation levels, even small delays in tax collections reduce the real value of tax revenues, which is one reason real tax revenues decline in hyperinflationary episodes, as discussed in Sargent (1983).

debt is now denominated in a unit of account that is worth less. Once we adjust the change in real debt for inflation using equation (6.5), we obtain

$$
\begin{aligned}
\frac{\hat{B}_t}{P_t} - \frac{\hat{B}_{t-1}}{P_{t-1}} =& \frac{D_t}{P_t} - \frac{\hat{B}_{t-1}}{P_{t-1}}\left(1 - \frac{P_{t-1}}{P_t}\right) = \\
& \frac{G_t - T_t}{P_t} + \left(\frac{P_{t-1}}{\beta P_{t-2}} - 1\right)\frac{\hat{B}_{t-1}}{P_t} - \frac{\hat{B}_{t-1}}{P_{t-1}}\left(1 - \frac{P_{t-1}}{P_t}\right) = \\
& \frac{G_t - T_t}{P_t} - \frac{\hat{B}_{t-1}}{P_{t-1}}\left(-\frac{P_{t-1}}{\beta P_{t-2}} \cdot \frac{P_{t-1}}{P_t} + \frac{P_{t-1}}{P_t} + 1 - \frac{P_{t-1}}{P_t}\right) = \\
& \frac{G_t - T_t}{P_t} - \frac{\hat{B}_{t-1}}{P_{t-1}}\left(1 - \frac{P_{t-1}}{\beta P_{t-2}} \cdot \frac{P_{t-1}}{P_t}\right).
\end{aligned}
\tag{6.7}
$$

In a constant inflation environment, with $P_t/P_{t-1} = P_{t-1}/P_{t-2}$, the change in the real value of debt becomes independent of inflation (assuming that the real primary deficit $(G_t - T_t)/P_t$ is independent of inflation). In this case, we obtain

$$
\frac{\hat{B}_t}{P_t} - \frac{\hat{B}_{t-1}}{P_{t-1}} = \frac{G_t - T_t}{P_t} + \frac{\hat{B}_{t-1}}{P_{t-1}}\left(\frac{1}{\beta} - 1\right):
$$

the change in the real value of debt is driven by the real primary deficit and by the product of real debt and $(1/\beta - 1)$, which is the real interest rate of our model economy. This confirms that our adjustment is having the intended effect: the taxpayers are only burdened by the payments to creditors that are due to the real interest rate being positive.

In a non-constant inflation environment, equation (6.7) is more complicated because of the mismatch in timing between the correction and the Fisher equation. This arises from our specific assumptions of the cash-in-advance economy with linear preferences for credit goods.[3] There is a general lesson here: in richer models, the Fisher equation does not hold period by period, and the relationship between measured deficits and inflation may be somewhat complicated. However, at longer horizons, where the Fisher equation tends to hold, deficits are mechanically swelled by inflation, and the correction that we introduced undoes that. When inflation is in steady state, this emerges precisely.

[3] If preferences were linear in cash goods instead, this issue would not arise.

6.4 Adjusting for real growth

The change in the real value of debt is informative of the additional resources that will needed in the future to repay the government's obligations. However, we are often interested in how those resources compare to the government's *ability* to repay them. This is quite similar to what happens with private debtors. A bank will be much more willing to extend a \$300,000 mortgage to a customer whose annual income is \$100,000 rather than to one whose income is just \$50,000. Greece's debt load is crushing for Greece, but the same real amount would amount to little if it were Germany, because Germany's economy is much bigger. It is thus common to look at the debt/GDP ratio. GDP acts as the annual income in our mortgage example.[4]

In this section, we will study the relationship between deficits and the change in the debt/GDP ratio. Define Y_t as a country's *nominal* GDP in period t. Mimicking what we did for real debt in equation (6.5), we get

$$\frac{\hat{B}_t}{Y_t} - \frac{\hat{B}_{t-1}}{Y_{t-1}} = \frac{\hat{B}_t}{Y_t} - \frac{\hat{B}_{t-1}}{Y_t} + \frac{\hat{B}_{t-1}}{Y_t} - \frac{\hat{B}_{t-1}}{Y_{t-1}} = \frac{D_t}{Y_t} - \frac{\hat{B}_{t-1}}{Y_{t-1}}\left(1 - \frac{Y_{t-1}}{Y_t}\right). \tag{6.8}$$

The change in the debt/GDP ratio is given by the sum of two terms: the deficit/GDP ratio, plus an adjustment that is the product of the initial debt/GDP ratio times a factor related to nominal GDP growth. A country that is growing fast can afford large deficits and yet retain the debt/GDP ratio under control.

Defining y_t as real GDP (Y_t/P_t), it is useful to note

$$\frac{Y_t}{Y_{t-1}} = \frac{y_t}{y_{t-1}} \cdot \frac{P_t}{P_{t-1}}: \tag{6.9}$$

nominal GDP growth may come from either real GDP growth or inflation. The effect of the two sources of growth on public finances will be different. As we observed in the previous section, inflation, if it is anticipated, increases deficits through its effect on interest payments and does

[4] As is the case for the annual income in the example, GDP is an imperfect measure of a country's ability to raise taxes and repay its obligations. A literature on debt capacity has emerged, with Reinhart and Rogoff being prominent exponents. While imperfect, GDP remains an excellent way to scale debt to get an idea of the burden that debt imposes on a country's taxpayers.

not contribute to debt relief. In contrast, real growth will help in making a country's debt more manageable (holding the stance of fiscal policy fixed).[5]

It is often stated that countries that successfully repaid their national debt did so mostly by "growing out" of their debt problem; this means that the real-growth factor from equations (6.8) and (6.9) was responsible for a significant fraction of the drop in their debt/GDP ratio.

As in the previous section, we can use a linear approximation of the adjustment factor in equation (6.8) when both real growth and price inflation are small (y_t/y_{t-1} and P_t/P_{t-1} close to 1). This yields

$$\frac{\hat{B}_{t-1}}{Y_{t-1}}\left(1-\frac{Y_{t-1}}{Y_t}\right)=\frac{\hat{B}_{t-1}}{Y_{t-1}}\left(1-\frac{y_{t-1}}{y_t}\cdot\frac{P_{t-1}}{P_t}\right)\approx\frac{\hat{B}_{t-1}}{Y_{t-1}}\left(\frac{y_t}{y_{t-1}}-1+\frac{P_t}{P_{t-1}}-1\right):$$

the adjustment is approximately the product of the initial level of debt times the sum of real growth and inflation.

6.5 An example: public finances in the United Kingdom

We illustrate the procedures discussed above using the public finance accounts of the United Kingdom in Fiscal Year 2011/12. My data source is the public sector finances table of the Office of National Statistics.

For this analysis, we need to look at the empirical counterpart to our model concepts. For "taxes," I used total current receipts from Sheet psf3b-1. As you can see from the breakdown, not all of the government receipts are taxes in reality. For "spending," I summed the total current expenditure, plus government investment, which is itself made of two pieces: depreciation plus net investment (Sheet psf3b-2). The deficit also appears on the same Sheet as "net borrowing." I computed net interest payments subtracting from interest in the current expenditure category

[5] In many economic models, faster growth may lead to higher real interest rates, which would then increase a country's interest payments. This link is somewhat muted in the data. Furthermore, with international capital mobility, the real interest rate will be to some extent driven by worldwide forces, so the link between growth and interest rates will be less tight even theoretically. It is also worth noting that fast growth typically helps also the primary balance: as examples, in a fast-growing economy, there is less demand for unemployment benefits, and pension payments are linked to salaries that are smaller than the currently prevailing ones.

the value of interest and dividends from Sheet psf3b-1 in the current receipt category. For debt, I used public sector net debt ex, column H, lower part of Sheet psf6b. "Ex" stands for excluding companies that were controlled by the government, but expected to be (re)privatized in the near future (e.g. Lloyds bank). One inconsistency of which you should be aware is that public sector net debt refers to all of the public sector, including Councils and entities such as Network Rail. Spending and receipts only refer to the central government.

From these sources, total government receipts (T_t in the model) were £537,821 million, and total government outlays (G_t) were £649,991 million. Net borrowing (the difference between the two) was £112,170 million, of which £38,490 million were interest payments (the rest was primary deficit).

To compute the adjustment factor for the change in real debt, I start from public sector net debt (B_{t-1}), which was £1,004,919 million at the end of Fiscal year 2010/2011. I then measure inflation between the first quarter of 2011 and the first quarter of 2012 (this roughly corresponds to Fiscal Year 2011/2012). In choosing my measure of inflation, I used the GDP deflator. The GDP deflator measures the change in prices of the goods *produced* in the United Kingdom. Since the tax capacity of the country is based on how much it produces, this seems a more natural choice than CPI inflation, which measures the change in price of the goods *consumed* in the United Kingdom. GDP deflator (as well as GDP measures) comes from the Quarterly National Accounts of the Office of National Statistics. GDP inflation was 2.01% between 2011Q1 and 2012Q1: $P_t/P_{t-1} = 1.0201$.

The deficit adjustment is thus

$$\frac{\hat{B}_{2011Q1}}{P_{2011Q1}}\left(1-\frac{P_{2011Q1}}{P_{2012Q1}}\right) = £1,004,919 \text{ million } \times\left(1-\frac{1}{1.0201}\right) \approx £19,800 \text{ million }. \tag{6.10}$$

Notice that, in going from the adjustment factor in equation (6.5) to equation (6.10), I set $P_{2011Q1} = 1$. This is a normalization that implies that the deficit adjustment is measured in 2011Q1 pounds.[6] Had we used the linear approximation, we would have obtained a very similar

[6]To be very precise, I should have taken into account that the deficit is measured in Fiscal Year 2011/2012 pounds (an average of the price level of the four quarters from 2011Q2 to 2012Q1). I should then have converted the adjustment in the same unit. This complication arises from the fact that we are computing the evolution of

answer:

$$\frac{\hat{B}_{2011Q1}}{P_{2011Q1}} \cdot \left(\frac{P_{2012Q1}}{P_{2011Q1}} - 1\right) = £1,004,919 \text{ million } \times .0201 \approx £20,200 \text{ million.}$$

At 2% inflation, the linear approximation is excellent. We conclude that the adjustment accounts for roughly half of the net interest payments: only half of those payments represented a real burden for the taxpayers. The adjustment is about 20% of the deficit, which shows that, even in a year of low inflation and high deficit, accounting for the effect of inflation on interest payments is important to ascertain a correct picture of the state of public finances.

Next, consider the evolution of the debt/GDP ratio. Nominal GDP in 2011Q1 (what we call Y_{t-1}) was £1,520,948 million at an annual rate.[7] Nominal growth rate between 2011Q1 and 2012Q1 was 2.57%: $Y_t/Y_{t-1} = 1.0257$. Of this, we already observed that 2.01% was due to inflation, whereas the rest was real growth. The adjustment term in the change in the debt/GDP ratio (equation (6.8)) is thus

$$\frac{\hat{B}_{2011Q1}}{Y_{2011Q1}}\left(1 - \frac{Y_{2011Q1}}{Y_{2012Q1}}\right) = \frac{1,004,919}{1,520,948} \times \left(1 - \frac{1}{1.0257}\right) \approx 0.0166 = 1.66\% \text{ of 2011Q1 GDP.}$$

This compares with the deficit/GDP ratio, which was[8]

$$D_{2011/12}/Y_{2011Q1} = 7.38\%.$$

Fiscal year 2011/12 was a bad year for UK finances, with a very high deficit/GDP ratio. While both inflation and growth were modest by historical standards, they nonetheless helped in slowing the growth in the debt/GDP measure.

debt not over one quarter, but over four quarters. However, since inflation was just 2% over the entire year, the conversion would change the answer by less than 2%, so I neglected this complication.

[7] The debt/GDP ratio is almost universally expressed as a fraction of *annual* GDP, so it is important to convert the GDP measure into an annual number. With seasonally adjusted series, this requires simply a multiplication by 4.

[8] As in the case of the price correction, it would be more precise to divide the deficit by GDP over the entire 2011/2012 fiscal year. Here too this would represent only a minor difference, since growth over the entire year was 2.57% and thus the resulting number would differ from our calculation by just a few basis points ($.0257 \cdot 7.38 \approx .19\%$).

6.6 Long-term debt

Up to now, our notes have assumed that all of the government debt is short term. When long-term debt is present, the change in value of the debt depends on one additional factor: the way in which long-term interest rates move from one period to the next.

Specifically, assume now that, at the beginning of period t, the government is not committed to a single repayment B_{t-1}, but rather to a schedule of future payments $B_{t-1\,s}$, with $s = 0, \ldots, T$.[9] The value of debt will now be

$$\mathcal{B}_t := \sum_{s=0}^{T} \rho_{ts} B_{t-1\,s},$$

where ρ_{ts} is the time-t price of a zero-coupon bond maturing in period $t + s$. Changes in ρ will represent a further source of movements in the value of debt. In this case, inflation surprises will act on the real value of debt and on debt/GDP through the adjustment factors of equations (6.5) and (6.8), but also because a higher inflation surprise will increase long-term interest rates and thus depress long-term bond prices and thereby the value of outstanding debt.

For further details on this, see Hall and Sargent (1997, 2011, 2014).

6.7 Conclusion

The government deficit paints a misleading picture of the true balance of government finances over time. Some interest payments, counted in deficit, just make up for loss in real value of debt and should not be counted in assessing the fiscal balance. Furthermore, when we are interested in the sustainability of a given debt position, a faster growing economy may be more easily able to keep its debt under control, because the tax base that will be called to service the debt will be growing fast too.

[9] $T + 1$ is the maximum debt maturity issued by the government. In the case of the United States, this is 30 years. In the case of the United Kingdom, it is infinite: the UK government issued consols, which are promises to perpetual recurring payments.

Chapter 7

Fiscal Consequences of Paying Interest on Reserves (joint with Todd Messer, originally printed in *Fiscal Studies*)

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