More on Social Value of Information

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Motivation

- We saw that public information can be detrimental to welfare in Morris and Shin
- There are other papers where private behavior is observationally equivalent, yet conclusions are different
- Goal: to better understand why this happens

Some philosophy

- Economics is predicated on revealed preference (see e.g. Gul and Pesendorfer, "The case for mindless economics")
- Utility function is simply a (mathematically useful) representation of a preference ordering
- Paper indirectly points out a challenging aspect of optimal policy:
  - Different models with the same private preference ordering embed different externalities
  - Optimal policy depends on externality
  - Need more sophisticated ways of eliciting preferences (or trusting our model)

The abstract problem

- Fundamental: θ
- Continuum of identical households
- Preferences:

$$U(k, K, \sigma_k^2, \theta) = \frac{1}{2} \begin{bmatrix} k & K & \theta \end{bmatrix} \begin{bmatrix} U_{kk} & U_{kK} & U_{k\theta} \\ U_{kK} & U_{KK} & U_{K\theta} \\ U_{k\theta} & U_{K\theta} & U_{\theta\theta} \end{bmatrix} \begin{bmatrix} k \\ K \\ \theta \end{bmatrix} + \frac{1}{2} U_{\sigma\sigma} \sigma_k^2$$

- k: individual action (used to be a_i); K: average action (used to be $\bar{a}$)
- Cross-sectional volatility σ_k^2 (may) enter as a pure externality, no effect on individual behavior
- Results from assuming continuous-agent limit of a quadratic loss across finite agents, with anonymity and symmetry

Assumptions on utility

- Normalized k so that first-order effect of k is 0. Also, first-order effect of θ is irrelevant (θ exogenous), so set to zero
- Assume first-order effect of K is zero. Not a normalization, but nothing to do with information processing: planner might wish households to use policy with different intercept.
- Concavity in the individual action: $U_{kk} < 0$
- Unique equilibrium: $-U_{kK}/U_{kk} < 1$ (own second derivative stronger than cross derivative with others' actions
- Concavity of the social planner problem: $U_{kk} + 2U_{kK} + U_{KK} < 0$, $U_{kk} + U_{\sigma\sigma} < 0$

Morris-Shin as a special case

- $U_{kk} = -2$
- $U_{k\theta} = 2(1-r)$
- $U_{kK} = 2r$
- $U_{\theta\theta} = -2(1-r)$
- $U_{KK} = -2r$
- $U_{K\theta} = 0$
- $U_{\sigma\sigma} = 2r$

Distributional assumptions

- Will work with normal distributions
- Uninformative prior on θ: infinite variance (0 precision)
- Public signal z, $z|\theta \sim N(\theta, 1/\beta_z)$
- Private signal x_i, $x|\theta \sim N(\theta, 1/\beta_x)$
- $x_i|\theta \perp z|\theta$, usual iid-like assumptions on x_i

Linear equilibrium: definition

Affine $k(x,z)$ such that

$$k(x,z) = \arg\max_{k'} E[U(k', K(\theta,z), \sigma_k^2(\theta,z), \theta)|x,z]$$

where

$$K(\theta,z) = \int_x k(x,z) dP(x|\theta)$$

and

$$\sigma_k^2(\theta,z) = \int_x [k(x,z) - K(\theta,z)]^2 dP(x|\theta)$$

Complete information benchmark

- Suppose θ is known (either $\beta_x = \infty$ or $\beta_z = \infty$)
- Guess $k = \kappa_0 + \kappa_1 \theta$
- Solve individual problem
- Substitute $K = \kappa_0 + \kappa_1 \theta$
- Compute fixed point, get $\kappa_0 = 0$, $\kappa_1 = -U_{k\theta}/(U_{kk} + U_{kK})$
- Morris-Shin: $\kappa_1 = 1$

Computing an equilibrium

- Household first-order condition

$$-U_{kk}k - E\left[U_{kK}K(\theta, z) + U_{k\theta}\theta | x, z\right] = 0$$

- Define strategic complementarity $\alpha := -U_{kK}/U_{kk}$ ($= r$ in MS)
- Get

$$k = E[(1-\alpha)\kappa_1\theta + \alpha K(\theta, z) | x, z]$$

- Guess $k = \tilde{\kappa}_0 + \kappa_x x + \kappa_z z$
- Get

$$\tilde{\kappa}_0 + \kappa_x x + \kappa_z z = \alpha\tilde{\kappa}_0 + [(1-\alpha)\kappa_1 + \alpha\kappa_x]\left[\frac{\beta_x}{\beta_x + \beta_z}\right]x$$
$$+ \left[(1-\alpha)\kappa_1\frac{\beta_z}{\beta_x + \beta_z} + \kappa_z\right]z$$

Overweighting

- Define

$$\gamma := \frac{\beta_z}{(1-\alpha)\beta_x + \beta_z}$$

- Solve fixed point: $\tilde{\kappa}_0 = 0$,

$$\kappa_x = (1-\gamma)\kappa_1, \qquad \kappa_z = \gamma\kappa_1$$

- Relative to single-agent problem, overweighting of z when $\alpha > 0$

Efficient use of information (planner problem)

- Planner has to respect separation of agents
- No information sharing (otherwise the problem is trivial)
- Planner can choose $k(x, z)$
- Planner problem:

$$\max_{k(x,z)} E[E[U(k(x,z), K(\theta,z), \sigma_k^2(\theta,z), \theta)|x,z]]$$

where

$$K(\theta,z) = \int_{x'} k(x',z)dP(x'|\theta)$$

and

$$\sigma_k^2(\theta,z) = \int_{x'} [k(x',z) - K(\theta,z)]^2 dP(x'|\theta)$$

- Individual takes $K(\theta, z)$ and $\sigma_k^2(\theta, z)$ as given, planner does not

Solving the planner problem: Part 1

- Objective function is quadratic, so the solution will be linear
- Can verify that constant terms are zero (because I killed first derivatives)
- Optimize directly over $\hat{\kappa}_x x + \hat{\kappa}_z z$
- $K(\theta, z) = \hat{\kappa}_x \theta + \hat{\kappa}_z z$
- $\sigma_k^2(\theta, z) = (\hat{\kappa}_x)^2 / \beta_x$

Solving the planner problem: Part 2

- $$\begin{bmatrix} k \\ K \\ \theta \end{bmatrix} = \begin{bmatrix} \hat{\kappa}_x + \hat{\kappa}_z & \hat{\kappa}_x & \hat{\kappa}_z \\ \hat{\kappa}_x + \hat{\kappa}_z & 0 & \hat{\kappa}_z \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ x - \theta \\ z - \theta \end{bmatrix}$$
- Substitute into objective function, take expected value
- Note 1: variance of θ is infinite, we'd better zero out the term in θ^2
- Note 2: all covariances are zero, can neglect off-diagonal terms of the ugly matrix that results after substitution

Solving the planner problem: Part 3

- To zero out variance of θ term, we need

$$(\kappa_x^*+\kappa_z^*)^2(U_{kk}+2U_{kK}+U_{KK})+2(\kappa_x^*+\kappa_z^*)(U_{k\theta}+U_{K\theta})+U_{\theta\theta}=0$$

- Quadratic equation that pins down $\kappa_x^* + \kappa_z^*$ ($= \kappa_1^*$ in the paper)
- MS: $\kappa_1^* = 1$
- Remainder of the problem:

$$\max_{\hat{\kappa}_x,\hat{\kappa}_z} \frac{(\hat{\kappa}_x)^2\left(U_{kk}+U_{\sigma\sigma}\right)}{\beta_x} + \frac{(\hat{\kappa}_z)^2\left(U_{kk}+2U_{kK}+U_{KK}\right)}{\beta_z}$$

subject to $\hat{\kappa}_x + \hat{\kappa}_z = \kappa_1^*$

- Define $\gamma^* := \kappa_z^*/\kappa_1^*$

Inspecting the economics

$$\gamma^* = \arg\max_{\hat{\gamma}} \frac{(1-\hat{\gamma})^2 (U_{kk} + U_{\sigma\sigma})}{\beta_x} + \frac{(\hat{\gamma})^2 (U_{kk} + 2U_{kK} + U_{KK})}{\beta_z}$$

- We assumed $U_{kk} + U_{\sigma\sigma} < 0$ and $U_{kk} + 2U_{kK} + U_{KK} < 0$
- $1 - \gamma^*$: Increases exposure to x noise, individual effect + externality from $U_{\sigma\sigma}$ (notice x noise vanishes in aggregate)
- γ^*: Increases exposure to z noise, increases aggregate volatility (externalities in play again)
- Note that U_{KK} and $U_{\sigma\sigma}$ encode pure externalities not identifiable with private behavior

Alternative representation

- Define

$$\alpha^* := 1 - \frac{U_{kk} + 2U_{kK} + U_{KK}}{U_{kk} + U_{\sigma\sigma}}$$

- Morris-Shin: $\alpha^* = 0$: no complementarities in social welfare
- Social planner problem is

$$\min_{\hat{\gamma}} \frac{(1-\hat{\gamma})^2}{\beta_x} + \frac{(1-\alpha^*)(\hat{\gamma})^2}{\beta_z}$$

- α^* is a measure of social complementarity, can be compared to equilibrium α
- Bigger $\alpha^* \implies$ bigger $\gamma^* \implies$ stronger response to public signals

Comparing equilibrium allocation and efficient allocation

- κ_1 vs κ_1^*: how strongly actions should respond to θ (even under full info)
- α vs. α^* (or γ vs. γ^*): how strongly they should respond to public vs private signals

A useful decomposition

- Consider $\theta - E[\theta|x, z]$
- Define $\beta := \beta_x + \beta_z$, precision of individual info about θ
- Define δ as correlation of information across people:

$$\delta = \text{Corr}\left(\theta - E[\theta|x, z], \theta - E[\theta|x', z]\right) = \frac{\beta_z}{\beta_x + \beta_z}$$

Economies with $\kappa_1 = \kappa_1^*$

Social loss is (proportional to)

$$\frac{(1-\gamma)^2}{\beta_x} + \frac{(1-\alpha^*)(\gamma)^2}{\beta_z}$$

Information in efficient economies

- Efficiency requires $\gamma = \gamma^*$
- Loss simplifies to

$$\frac{1-\alpha^*}{(1-\alpha^*)\beta_x + \beta_z} = \frac{(1-\alpha^*)(1+\delta)}{\beta[1-\alpha^*+\delta]}$$

- More precision of either type always good
- Higher δ good if $\alpha^* > 0$: more complementarity, want common signals

Information in economies with $\kappa_1 = \kappa_1^*$ but $\gamma \neq \gamma^*$

- Social loss is

$$\frac{(1-\gamma)^2}{\beta_x} + \frac{(1-\alpha^*)(\gamma)^2}{\beta_z} = \frac{1+\delta}{\beta}\left[(1-\gamma)^2(1+\delta)+\gamma^2\delta\right]$$

- Higher β always good (increase precision of signals proportionately)
- In equilibrium $\gamma = \beta_z/[(1-\alpha)\beta_x + \beta_z] = \delta/(1-\alpha+\delta)$ and loss is

$$\frac{(1-\alpha)^2\beta_x + (1-\alpha^*)\beta_z}{[(1-\alpha)\beta + \beta_z]^2} = \frac{1+\delta}{\beta(1-\alpha+\delta)^2}[(1-\alpha^2)+\delta(1-\alpha^*)]$$

- Effect of δ is ambiguous

Saying a bit more

- We already know that higher δ is good (bad) for $\alpha = \alpha^* > (<)0$
- If α is not too large, the derivative of loss wrt δ is decreasing in α^*
- Take $\alpha > 0$. When $\alpha^* > \alpha$, higher δ is even better
- With $\alpha < 0$, when $\alpha^* < \alpha$, higher δ is even worse

What if $\kappa_1 \neq \kappa_1^*$?

- In our case, infinite social loss, due to noninformative prior
- In the paper, a novel first-order effect shows up
- This new effect is proportional to

$$\frac{\kappa_1(\kappa_1^* - \kappa_1)}{\beta(1-\alpha(1-\delta))}$$

- Higher precision can be bad if κ_1 and $\kappa_1^* - \kappa_1$ have opposite signs
- Intuition: want to dampen household response, noisy information will do it

Thinking about policy

- Let there be a government
- The government cannot communicate info in real time
- Gov't can only set taxes to be paid at the end of the period
- Set tax policy as

$$T(k,K,\theta) = \bar{T} + \frac{1}{2}\begin{bmatrix} k & K & \theta \end{bmatrix}\begin{bmatrix} T_{kk} & T_{kK} & T_{k\theta} \\ T_{kK} & T_{KK} & T_{K\theta} \\ T_{k\theta} & T_{K\theta} & T_{\theta\theta} \end{bmatrix}\begin{bmatrix} k \\ K \\ \theta \end{bmatrix}$$

Notes on tax function

$$T(k,K,\theta) = \bar{T} + \frac{1}{2}\begin{bmatrix} k & K & \theta \end{bmatrix}\begin{bmatrix} T_{kk} & T_{kK} & T_{k\theta} \\ T_{kK} & T_{KK} & T_{K\theta} \\ T_{k\theta} & T_{K\theta} & T_{\theta\theta} \end{bmatrix}\begin{bmatrix} k \\ K \\ \theta \end{bmatrix}$$

- Linear utility in taxes (transferable utility)
- Specification assumes θ is observed ex post; not needed
- We could have linear terms, do not need them given our assumptions

Balance-budget requirement

- $$\bar{T}+\frac{1}{2}\begin{bmatrix} K & K & \theta \end{bmatrix}\begin{bmatrix} T_{kk} & T_{kK} & T_{k\theta} \\ T_{kK} & T_{KK} & T_{K\theta} \\ T_{k\theta} & T_{K\theta} & T_{\theta\theta} \end{bmatrix}\begin{bmatrix} K \\ K \\ \theta \end{bmatrix}+\frac{T_{kk}}{2}\sigma_k^2 \equiv 0$$
- Set $\bar{T} = -\frac{T_{kk}}{2}\sigma_k^2$
- Need $T_{\theta\theta} = 0$
- $$T_{kk} + 2T_{kK} + T_{KK} = 0 \implies \text{ get } T_{KK}$$
- $$T_{k\theta} + T_{K\theta} = 0 \implies \text{ get } T_{K\theta}$$

Modified household problem

- Household problem is the same as before, except that U_{kk} is replaced by $U_{kk} + T_{kk}$ (and similarly for all other terms)
- To get efficiency, we need $\kappa_1 = \kappa_1^*$ and $\alpha = \alpha^*$
- $$\kappa_1^g = -\frac{U_{k\theta} + T_{k\theta}}{U_{kk} + U_{kK} + T_{kk} + T_{kK}}$$
- $$\alpha^g = -\frac{U_{kK} + T_{kK}}{U_{kk} + T_{kk}}$$
- Use T_{kk}, T_{kK}, $T_{k\theta}$

Morris-Shin application

- $$\frac{-2(1-r)+T_{k\theta}}{-2(1-r)+T_{kk}+T_{kK}}=1 \Longrightarrow T_{k\theta}=T_{kk}+T_{kK}$$
- $$-\frac{r+T_{kK}}{-2+T_{kk}}=0 \Longrightarrow T_{kK}=-r$$
- Can use (for example) $T_{k\theta}=r$ and $T_{kk}=0$, or $T_{k\theta}=0$ and $T_{kk}=r$