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Determinacy without the Taylor Principle

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May 2, 2024

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Weakness in Equilibrium Determinacy

- Whether we use the Taylor principle or the FTPL...
- ... determinacy is about expectations of events far in the future

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Weakness in Equilibrium Determinacy

- Whether we use the Taylor principle or the FTPL...
- ... determinacy is about expectations of events far in the future
- What strategies will be credible that far in advance?
- Will people understand those strategies?
- Will "simple" solutions be focal points?
- Big literature on bounded rationality (limited attention, level-k thinking, "sparsity," ...)
- Today: imperfect recall



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Punchline

- With imperfect recall, the "minimum state variable" solution prevails
- Sunspots do not arise
- Interest rate rules do not have to satisfy the Taylor principle

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Punchline

- With imperfect recall, the "minimum state variable" solution prevails
- Sunspots do not arise
- Interest rate rules do not have to satisfy the Taylor principle
- Departures from perfect recall are very slight...
- ... but carefully placed in the right spots

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The Model: IS/Euler Equation

- Stripped-down 3-equation NK model
- Loglinearized (look for linear equilibria)

$$c_t = -\sigma(i_t - \tilde{E}_t \pi_{t+1}) + \tilde{E}_t c_{t+1} + \sigma \rho_t$$

• Expectation gets a tilde because we will play with the information set

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Phillips Curve

• Phillips curve:

$$\pi_t = \kappa(c_t + \xi_t)$$

- No forward-looking component in the Phillips curve
- Mostly for simplicity
- Sidesteps two big complications:
 - Whose expectations? Households? Firms?
 - What are you forming expectations about? (In learning, Euler equation vs. deeper learning, see Preston, IJCB, 2005)

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Taylor Rule

$$i_t = \phi \pi_t + z_t$$

• No output (consumption): purely for simplicity

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Main Difference Equation

• Substitute Taylor rule + Phillips curve into Euler

• Get

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$$c_t = \delta \tilde{E}_t c_{t+1} + \theta_t$$

• θ_t : combination of all the shocks

$$\delta = \frac{1 + \sigma \kappa}{1 + \phi \sigma \kappa}$$

Two possibilities:

- $\phi > 1 \Longrightarrow \delta < 1$
- $\phi < 1 \Longrightarrow \delta > 1$

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Shock Processes

- For tractability, work within linear-Gaussian world
- Other than that, very flexible structure
- Fundamentals:

$$\theta_t = q \cdot x_t$$

$$x_t = Rx_{t-1} + \epsilon_t$$

Stationarity: all eigenvalues are less than 1 in modulus

$$\epsilon_t \sim N(0, \Sigma)$$

• Sunspot (i.i.d.):

$$\eta_t \sim N(0,1)$$

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(Stationary Linear) Equilibrium

A stochastic process $\{c_t\}_{t=0}^{\infty}$, adapted to $\{x_s, \eta_s\}_{s=-\infty}^t$, that satisfies

$$c_t = \delta \tilde{E}_t c_{t+1} + \theta_t,$$

$$c_t = \sum_{k=0}^{\infty} \left[a_k \eta_{t-k} + \gamma_k \cdot x_{t-k} \right],$$

the information restrictions that we will impose on \tilde{E}_t , and $\operatorname{Var}(c_t) < \infty$.

• Note: Stationarity rules out explosive equilibria.

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Full-Info Benchmark

- Information set: the entire history of both shocks and endogenous variables.
- Note: shocks are enough. Any equilibrium can be represented as a function of shocks...
- Remember that past endogenous variables are linear functions of shocks (and possibly other past variables, recursive substitution)

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Solving by Guess and Verify

$$c_{t} = \delta E_{t} c_{t+1} + \theta_{t},$$

$$c_{t} = \sum_{k=0}^{\infty} \left[a_{k} \eta_{t-k} + \gamma_{k} \cdot x_{t-k} \right],$$

$$E_{t} x_{t+1} = R x_{t}$$

$$\Longrightarrow$$

$$\sum_{k=0}^{\infty} \left[a_{k} \eta_{t-k} + \gamma_{k} \cdot x_{t-k} \right] = \delta \sum_{k=0}^{\infty} \left[a_{k+1} \eta_{t-k} + \gamma_{k+1} \cdot x_{t-k} \right] + \left(q' + \delta \gamma'_{0} R \right) x_{t}$$

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Solution when $\delta < 1$ ($\phi > 1$)

Match coefficients:

$$a_{k} = \delta a_{k+1}$$

$$\gamma_{k} = \delta \gamma_{k+1} k > 0$$

$$\gamma'_{0} = \delta \gamma'_{1} + q' + \delta \gamma'_{0} R$$

$$\Longrightarrow$$

$$a_{k+1} = (1/\delta) a_{k} \Longrightarrow a_{k} \equiv 0$$

$$\gamma_{k+1} = (1/\delta) \gamma_{k}, k \ge 1 \Longrightarrow \gamma_{k} \equiv 0 k \ge 1$$

$$\gamma'_{0} = q' (I - \delta R)^{-1}$$

Unique stationary solution, no sunspots

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Solution when $\delta > 1$ ($\phi < 1$)

• Previous solution ("MSV") still works:

$$\gamma_0' = q'(I - \delta R)^{-1}, \gamma_k \equiv 0, k \ge 1, a_k \equiv 0, k \ge 0$$

• However, now we can add to it any arbitrary initial condition

 $(\bar{a}_0, \bar{\gamma}_0)$

and set

$$ar{a}_k = \delta^{-k} ar{a}_0$$

 $ar{\gamma}_k = \delta^{-k} ar{\gamma}_0$

Sunspot equilibria, indeterminate response to shocks

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The Fun Begins: Imperfect Information

- A fraction $\lambda(1-\lambda)^k$ of people remember the history only up to period t-k
- Note: Agents do not remember $\{c_{t-k}\}_{k>0}$.

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Main Result Proposition 2

Regardless of δ , the (locally) unique equilibrium is the MSV equilibrium.

- Two pieces:
 - The MSV is still an equilibrium
 - Nothing else is

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The MSV is still an equilibrium

$$c_t = q'(I - \delta R)^{-1} x_t$$

Notes:

- Everybody knows x_t, so c_t is measurable wrt info of the private sector
- x_t is a sufficient statistic for forecasting x_{t+1} and hence c_{t+1} , so

$$\tilde{E}_t c_{t+1} = q' (I - \delta R)^{-1} \tilde{E}_t x_{t+1} = q' (I - \delta R)^{-1} E_t x_{t+1}$$

 \Longrightarrow Euler equation holds as before

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There are no other Equilibria - proof for i.i.d. case Try guess and verify again:

$$c_{t} = \delta \tilde{E}_{t} c_{t+1} + \theta_{t},$$

$$\tilde{E}_{t} c_{t+1} = \sum_{k=0}^{\infty} \left[a_{k} \tilde{E}_{t} \eta_{t+1-k} + \gamma_{k} \cdot \tilde{E}_{t} x_{t+1-k} \right],$$

$$\tilde{E}_{t} \eta_{t-k} = (1-\lambda)^{k} \eta_{t-k}$$

$$\tilde{E}_{t} x_{t-k} = (1-\lambda)^{k} x_{t-k}$$

$$\Longrightarrow$$

$$\sum_{k=0}^{\infty} \left[a_{k} \eta_{t-k} + \gamma_{k} \cdot x_{t-k} \right]$$

$$= \delta \sum_{k=0}^{\infty} \left[a_{k+1} (1-\lambda)^{k} \eta_{t-k} + (1-\lambda)^{k} \gamma_{k+1} \cdot x_{t-k} \right] + q \cdot x_{t}$$

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Solution, matching coefficients again

Match coefficients:

$$egin{aligned} &a_k = \delta(1-\lambda)^k a_{k+1} \ &\gamma_k = \delta(1-\lambda)^k \gamma_{k+1}, k > 0 \ &\gamma_0 = \delta \gamma_1 + q \end{aligned}$$

• For k large, $\delta(1-\lambda)^k < 1$: no sunspots, only MSV

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$$a_k\equiv 0$$
, $\gamma_0=q$, $\gamma_k\equiv 0, k>0$

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Complications out of i.i.d. case

- Need to compute $E_t x_{t-k}$ for people that do not remember that far back
- Filtering problem
- Algebra a lot more involved, but intuition carries over unchanged

Is it innocuous to assume only knowledge of shocks?

- NO!
- Suppose people remember perfectly $c_{t-1}, x_{t-1}, \eta_{t-1}$
- Keep i.i.d. assumption

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Cranking out the Algebra

• Start from the guess:

$$\begin{aligned} c_t &= q \cdot x_t + \sum_{k=0}^{\infty} \delta^{-k} \left[\bar{a}_0 \eta_{t-k} + \bar{\gamma}_0 \cdot x_{t-k} \right] \\ &= q \cdot x_t + \delta^{-1} (c_{t-1} - q \cdot x_{t-1}) + \bar{a}_0 \eta_t + \bar{\gamma}_0 \cdot x_t \end{aligned}$$

• Now everybody who is forming expectations knows c_t , x_t :

$$\tilde{E}_t c_{t+1} = \delta^{-1} (c_t - q \cdot x_t)$$

• Get the same solution as full info, because (c_t, x_t) are a sufficient statistic

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Does it change of a few people forget?

- Suppose only a fraction 1λ of people remember $c_{t-1}, x_{t-1}, \eta_{t-1}$
- Guess a solution of the form

$$egin{aligned} c_t &= q\cdot x_t + \sum_{k=0}^\infty \psi^{-k} [ar{a}_0\eta_{t-k} + ar{\gamma}_0\cdot x_{t-k}] \ & q\cdot x_t + \psi^{-1} (c_{t-1} - q\cdot x_{t-1}) + ar{a}_0\eta_t + ar{\gamma}_0 x_t \end{aligned}$$

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New algebra

$$ilde{E}_t c_{t+1} = \psi^{-1} (c_t - q \cdot x_t) (1 - \lambda)$$

• Substitute into difference equation:

$$c_t = \delta(1-\lambda)\psi^{-1}(c_t - q \cdot x_t) + q \cdot x_t$$

• As long as
$$\psi = \delta(1-\lambda)$$
, OK

- If $\delta>1$ and $\lambda\approx$ 0, $\psi>1$
- Indeterminacy remains

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Breaking the result with more (small) noise

- Suppose there is a fundamental shock ζ_t (arbitrarily small) that is completely forgotten at t + 1
- (Retain i.i.d.) Normalize the MSV solution to

$$c_t = q \cdot x_t + \zeta_t$$

Suppose we try to represent solution as

$$c_t = q \cdot x_t + \zeta_t + \delta^{-1}(c_{t-1} - q \cdot x_{t-1} - \zeta_{t-1}) + \bar{a}_0\eta_t + \bar{\gamma}_0 \cdot x_t$$

- Problem: if c_{t-1} depends on sunspots, c_{t-1} ζ_{t-1} is not measurable wrt info at time t
- Proposition 4 builds on this.

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Other loose ends filled by the paper

- Full microfoundations \neq Euler equation
- Need longer-dated expectations
- Results go through

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What have we learned?

- Sunspot equilibria require a lot of coordination
- Even a bit of disruption unravels them
- You need to be careful where that disruption occurs
- · Heterogeneity plays an important role in this

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Thinking about the Real World

- "Inflation" is a fairly abstract object
- We all consume different baskets
- We are exposed to different prices
- When the only reason I respond to η_t is that you respond, coordination will be challenging
- Need some "fundamental" push