Determinacy without the Taylor Principle

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Weakness in Equilibrium Determinacy

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Weakness in Equilibrium Determinacy

- Whether we use the Taylor principle or the FTPL...
- ... determinacy is about expectations of events far in the future
- What strategies will be credible that far in advance?
- Will people understand those strategies?
- Will "simple" solutions be focal points?
- Big literature on bounded rationality (limited attention, level-k thinking, "sparsity," ...)
- Today: imperfect recall

Punchline

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- Sunspots do not arise
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Punchline

- With imperfect recall, the "minimum state variable" solution prevails
- Sunspots do not arise
- Interest rate rules do not have to satisfy the Taylor principle
- Departures from perfect recall are very slight...
- ... but carefully placed in the right spots

The Model: IS/Euler Equation

- Stripped-down 3-equation NK model
- Loglinearized (look for linear equilibria)
- $$c_t = -\sigma(i_t - \tilde{E}_t \pi_{t+1}) + \tilde{E}_t c_{t+1} + \sigma \rho_t$$
- Expectation gets a tilde because we will play with the information set

Phillips Curve

- Phillips curve:

$$\pi_t = \kappa(c_t + \xi_t)$$

- No forward-looking component in the Phillips curve
- Mostly for simplicity
- Sidesteps two big complications:
    - Whose expectations? Households? Firms?
    - What are you forming expectations about? (In learning, Euler equation vs. deeper learning, see Preston, IJCB, 2005)

Taylor Rule

$$i_t = \phi \pi_t + z_t$$

- No output (consumption): purely for simplicity

Main Difference Equation

- Substitute Taylor rule + Phillips curve into Euler
- Get

$$c_t = \delta \tilde{E}_t c_{t+1} + \theta_t$$

- θ_t: combination of all the shocks
- $$\delta = \frac{1 + \sigma\kappa}{1 + \phi\sigma\kappa}$$

  Two possibilities:
  - $\phi > 1 \Longrightarrow \delta < 1$
  - $\phi < 1 \Longrightarrow \delta > 1$

Shock Processes

- For tractability, work within linear-Gaussian world
- Other than that, very flexible structure
- Fundamentals:

$$\theta_t = q \cdot x_t$$

$$x_t = Rx_{t-1} + \epsilon_t$$

Stationarity: all eigenvalues are less than 1 in modulus

$$\epsilon_t \sim N(0, \Sigma)$$

- Sunspot (i.i.d.):

$$\eta_t \sim N(0, 1)$$

(Stationary Linear) Equilibrium

A stochastic process $\{c_t\}_{t=0}^{\infty}$, adapted to $\{x_s, \eta_s\}_{s=-\infty}^{t}$, that satisfies

$$c_t = \delta \tilde{E}_t c_{t+1} + \theta_t,$$

$$c_t = \sum_{k=0}^{\infty} \left[a_k \eta_{t-k} + \gamma_k \cdot x_{t-k} \right],$$

the information restrictions that we will impose on $\tilde{E}_t$, and $\mathrm{Var}(c_t) < \infty$.

- Note: Stationarity rules out explosive equilibria.

Full-Info Benchmark

- Information set: the entire history of both shocks and endogenous variables.
- Note: shocks are enough. Any equilibrium can be represented as a function of shocks...
- Remember that past endogenous variables are linear functions of shocks (and possibly other past variables, recursive substitution)

Solving by Guess and Verify

$$c_t = \delta E_t c_{t+1} + \theta_t,$$

$$c_t = \sum_{k=0}^{\infty} \left[a_k \eta_{t-k} + \gamma_k \cdot x_{t-k}\right],$$

$$E_t x_{t+1} = R x_t$$

$$\Longrightarrow$$

$$\sum_{k=0}^{\infty} \left[a_k \eta_{t-k} + \gamma_k \cdot x_{t-k}\right] = \delta \sum_{k=0}^{\infty} \left[a_{k+1} \eta_{t-k} + \gamma_{k+1} \cdot x_{t-k}\right] + (q' + \delta \gamma_0' R) x_t$$

Solution when $\delta < 1$ ($\phi > 1$)

Match coefficients:

$$a_k = \delta a_{k+1}$$

$$\gamma_k = \delta \gamma_{k+1} k > 0$$

$$\gamma_0' = \delta \gamma_1' + q' + \delta \gamma_0' R$$

$$\Longrightarrow$$

$$a_{k+1} = (1/\delta) a_k \Longrightarrow a_k \equiv 0$$

$$\gamma_{k+1} = (1/\delta)\gamma_k, k \geq 1 \Longrightarrow \gamma_k \equiv 0 k \geq 1$$

$$\gamma_0' = q'(I - \delta R)^{-1}$$

Unique stationary solution, no sunspots

Solution when $\delta > 1$ ($\phi < 1$)

- Previous solution ("MSV") still works:

$$\gamma_0' = q'(I - \delta R)^{-1}, \gamma_k \equiv 0, k \geq 1, a_k \equiv 0, k \geq 0$$

- However, now we can add to it any arbitrary initial condition

$$(\bar{a}_0, \bar{\gamma}_0)$$

  and set

$$\bar{a}_k = \delta^{-k} \bar{a}_0$$

$$\bar{\gamma}_k = \delta^{-k} \bar{\gamma}_0$$

- Sunspot equilibria, indeterminate response to shocks

The Fun Begins: Imperfect Information

- A fraction $\lambda(1-\lambda)^k$ of people remember the history only up to period $t-k$
- Note: Agents do not remember $\{c_{t-k}\}_{k>0}$.

Main Result

Proposition 2

Regardless of δ, the (locally) unique equilibrium is the MSV equilibrium.

- Two pieces:
  - The MSV is still an equilibrium
  - Nothing else is

The MSV is still an equilibrium

$$c_t = q'(I - \delta R)^{-1} x_t$$

Notes:

- Everybody knows x_t, so c_t is measurable wrt info of the private sector
- x_t is a sufficient statistic for forecasting x_{t+1} and hence c_{t+1}, so

$$\tilde{E}_t c_{t+1} = q'(I - \delta R)^{-1} \tilde{E}_t x_{t+1} = q'(I - \delta R)^{-1} E_t x_{t+1}$$

$\implies$ Euler equation holds as before

There are no other Equilibria - proof for i.i.d. case

Try guess and verify again:

$$c_t = \delta \tilde{E}_t c_{t+1} + \theta_t,$$

$$\tilde{E}_t c_{t+1} = \sum_{k=0}^{\infty} \left[a_k \tilde{E}_t \eta_{t+1-k} + \gamma_k \cdot \tilde{E}_t x_{t+1-k} \right],$$

$$\tilde{E}_t \eta_{t-k} = (1-\lambda)^k \eta_{t-k}$$

$$\tilde{E}_t x_{t-k} = (1-\lambda)^k x_{t-k}$$

$$\Longrightarrow$$

$$\sum_{k=0}^{\infty} [a_k \eta_{t-k} + \gamma_k \cdot x_{t-k}]$$

$$= \delta \sum_{k=0}^{\infty} \left[a_{k+1} (1-\lambda)^k \eta_{t-k} + (1-\lambda)^k \gamma_{k+1} \cdot x_{t-k} \right] + q \cdot x_t$$

Solution, matching coefficients again

Match coefficients:

$$a_k = \delta(1-\lambda)^k a_{k+1}$$

$$\gamma_k = \delta(1-\lambda)^k \gamma_{k+1}, k > 0$$

$$\gamma_0 = \delta\gamma_1 + q$$

- For k large, $\delta(1-\lambda)^k < 1$: no sunspots, only MSV
- $a_k \equiv 0$, $\gamma_0 = q$, $\gamma_k \equiv 0, k > 0$

Complications out of i.i.d. case

- Need to compute $E_t x_{t-k}$ for people that do not remember that far back
- Filtering problem
- Algebra a lot more involved, but intuition carries over unchanged

Is it innocuous to assume only knowledge of shocks?

- NO!
- Suppose people remember perfectly $c_{t-1}, x_{t-1}, \eta_{t-1}$
- Keep i.i.d. assumption

Cranking out the Algebra

- Start from the guess:

$$\begin{aligned} c_t &= q \cdot x_t + \sum_{k=0}^{\infty} \delta^{-k} \left[\bar{a}_0 \eta_{t-k} + \bar{\gamma}_0 \cdot x_{t-k} \right] \\ &= q \cdot x_t + \delta^{-1}(c_{t-1} - q \cdot x_{t-1}) + \bar{a}_0 \eta_t + \bar{\gamma}_0 \cdot x_t \end{aligned}$$

- Now everybody who is forming expectations knows c_t, x_t:

$$\tilde{E}_t c_{t+1} = \delta^{-1}(c_t - q \cdot x_t)$$

- Get the same solution as full info, because (c_t, x_t) are a sufficient statistic

Does it change of a few people forget?

- Suppose only a fraction $1 - \lambda$ of people remember $c_{t-1}, x_{t-1}, \eta_{t-1}$
- Guess a solution of the form

$$c_t = q \cdot x_t + \sum_{k=0}^{\infty} \psi^k [\bar{a}_0 \eta_{t-k} + \bar{\gamma}_0 \cdot x_{t-k}]$$
$$q \cdot x_t + \psi^{-1}(c_{t-1} - q \cdot x_{t-1}) + \bar{a}_0 \eta_t + \bar{\gamma}_0 x_t$$

New algebra

- $$\tilde{E}_t c_{t+1} = \psi^{-1}(c_t - q \cdot x_t)(1 - \lambda)$$
- Substitute into difference equation:
$$c_t = \delta(1 - \lambda)\psi^{-1}(c_t - q \cdot x_t) + q \cdot x_t$$
- As long as $\psi = \delta(1 - \lambda)$, OK
- If $\delta > 1$ and $\lambda \approx 0$, $\psi > 1$
- Indeterminacy remains

Breaking the result with more (small) noise

- Suppose there is a fundamental shock ζ_t (arbitrarily small) that is completely forgotten at $t+1$
- (Retain i.i.d.) Normalize the MSV solution to

$$c_t = q \cdot x_t + \zeta_t$$

- Suppose we try to represent solution as

$$c_t = q \cdot x_t + \zeta_t + \delta^{-1}(c_{t-1} - q \cdot x_{t-1} - \zeta_{t-1}) + \bar{a}_0 \eta_t + \bar{\gamma}_0 \cdot x_t$$

- Problem: if c_{t-1} depends on sunspots, $c_{t-1} - \zeta_{t-1}$ is not measurable wrt info at time t
- Proposition 4 builds on this.

Other loose ends filled by the paper

- Full microfoundations $\neq$ Euler equation
- Need longer-dated expectations
- Results go through

What have we learned?

- Sunspot equilibria require a lot of coordination
- Even a bit of disruption unravels them
- You need to be careful where that disruption occurs
- Heterogeneity plays an important role in this

Thinking about the Real World

- "Inflation" is a fairly abstract object
- We all consume different baskets
- We are exposed to different prices
- When the only reason I respond to η_t is that you respond, coordination will be challenging
- Need some "fundamental" push