

Forward Guidance: Communication, Commitment, or Both?

Online Appendix

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1 Verification of the Numerical Equilibria of Section 3

We substitute the numerical values of the parameters into equations (8), (9), and (17) to obtain the static babbling equilibrium. The parameter values are $k = 0.01$, $\pi_t^* \equiv 0.02$, $\beta = 0.96$, $\alpha = 1$, and $\lambda = 40$. By assumption, y_t^* is common knowledge, so $z_t^* = E(z_t^*|\mathcal{F}_t)$. We obtain

$$z_t = z_t^* + 16.$$

Using (8) and (9), we get

$$\pi_t = \pi_t^* + (z_t - z_t^*)/\lambda = 0.42.$$

Thus, if household future expectations are independent of the current behavior of the government, the equilibrium will feature 42% inflation.

Next, we retain the assumption that the government babbles and we verify that the threat of permanently reverting to 42% inflation provides a sufficient incentive for the government to implement 2% inflation.

The CB loss under permanent 2% inflation is $k^2 = .0001$. If the CB deviates from 2%, then the equilibrium implies that household expectations will be 42% independently of any future action. Hence, the best alternative for the government is to play the static best one-shot response to 2% expected inflation, followed by 42% in all subsequent periods. The static best response is given by equation (16). Using $\pi_t^e = 0.02$ and appropriately substituting equations (8), (9), and (10), we obtain that the best one-shot deviation is

$$\pi_t = \frac{\lambda^2}{\alpha + \lambda^2} \pi_t^e + \frac{\lambda}{\alpha + \lambda^2} k + \frac{\alpha}{\alpha + \lambda^2} \pi_t^* = 0.02025.$$

Using (1) and (2), the one-period loss incurred by the government by deviating is $6.25 \cdot 10^{-8}$. The one-period loss incurred by the government in each subsequent period is 0.1601. We have

$$(1 - 0.96) \cdot 6.25 \cdot 10^{-8} + 0.96 \cdot 0.1601 > 0.0001,$$

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which verifies that the CB has no incentive to deviate. This completes the proof that the strategy profile with no messages described in the text is an equilibrium.

Next, consider the equilibrium with messages. The continuation after any deviation is the same as described above. Before any deviation has taken place, the household strategy profile implies that they believe any message from the CB, as long as the message announces $m_t < 0.42$. The government has thus two options:

- Send a message m_t and carry through with the expected inflation, $\pi_t = m_t$. In this case, households will again believe the CB message in the subsequent period.
- Send a message m_t and deviate, setting the best response to the households' expectations of m_t . In this case, the continuation strategies imply permanent reversion to $\pi_t = 0.42$ from the next period on.

If the government opts for reporting truthfully its future action, $\pi_t^e = \pi_t$ and the best option for the government is to implement $m_t = \pi_t = \pi_t^* = 0.02$. This choice maintains credibility and the loss is 0.0001, as in the case of the best babbling equilibrium.

If the government chooses to deviate, it can attain a loss of zero in the first period of a deviation, by announcing $m_t = \pi^* - k/\lambda = 1.975\%$ and implementing $\pi_t = \pi_t^*$. However, from then on households will expect the static babbling equilibrium, and the CB best response will be to follow it. Since

$$0.96 \cdot 0.1601 > 0.0001,$$

the CB finds it optimal to announce 2% and truthfully implement it. QED.